A MANUAL OF PRACTICAL PHYSICS

H. E. HADLEY

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TORONTO

A MANUAL OF PRACTICAL PHYSICS

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MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON

1916

GLASGOW: PRINTED AT THE UNIVERSITY PRESS
BY ROBERT MACLEHOSE AND CO. LTD.

PREFACE

THOUGH many good manuals of elementary practical physics are available for general use in Secondary Schools, there are few of a reasonable size and moderate price suitable for students working to the standard of the pass Intermediate examinations of various British universities. Repeated requests for a course to correspond with this need are responsible for the preparation of the present laboratory guide. It is not suggested that the book contains all that such Intermediate examinations may demand as regards experimental work; nevertheless, the student who has performed most of the experiments described, and understands the principles taught by them, should find himself able to undertake any ordinary practical exercise in elementary physics.

Apart from examination requirements, the scope and substance of the course are such as may be followed profitably by those senior students in Secondary Schools who propose to specialise in science.

It is not assumed that students of Intermediate standard will require to carry out all the experiments described, as the majority of them will have taken already a preliminary course in physical measurements. But, as such previous work must vary in different cases, the course here described is complete in itself and includes a brief treatment of preliminary measurements, which may perhaps serve a useful purpose as revision exercises.

A number of the experiments are preceded by a brief reference to the theory of the principles involved. Most teachers are aware that even when the laboratory work is taken concurrently with a course of lectures, students frequently approach an experiment with an ill-defined notion of the principles which it is intended to

demonstrate. It may be held that the student ought not to be given so much guidance in a laboratory text-book, and that the training is more permanently useful when the student has to solve his own difficulties and to discover principles for himself. Experience shows that this latter procedure is possible only when the class is small and when more time is available than can usually be given to the subject. Also, while the ideal method is practicable with students of exceptionally good mental equipment, it fails entirely with the less fortunate majority. It is on behalf of this majority also that, as guidance in the style of recording observations, a number of the experiments are followed by tabulated data and deductions obtained in a laboratory by means of apparatus identical with that described in the text.

It will be noticed that the apparatus required by the course is of the simplest possible character. Elaborate and expensive instruments do not add to the educative value of an experiment, nor are they available in many laboratories; but even without them a course of this standard may be undertaken successfully.

At the end of the volume a collection is given of numerical and experimental exercises, many of them from examination papers; and thanks are due to the University of London for permission to include a few typical questions from its papers. The numerical exercises will help the student to calculate his own results with greater facility; and the other questions may be useful as additional experiments to test whether the student clearly understands the principles demonstrated in the text.

The author is grateful to Prof. R. A. Gregory and to Mr. A. T. Simmons for their experienced advice in the preparation of the course and in its passage through the Press, and for permission to make use of experiments in their *Exercises in Practical Physics* and other books of which they are joint authors. Their assistance, so willingly given, has removed many difficulties which the author unaided could not hope to have solved satisfactorily.

H. E. HADLEY.

KIDDERMINSTER,
March, 1916.

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CHAPTER I.

MEASUREMENT OF LENGTH, ANGLE AND TIME.

LENGTH.

Use of scales.—Straight lines are measured by means of wood or metal scales, the edges of which are divided either into British or into Metric units. Before using a scale it is necessary to observe into what fractions the units are divided; thus, if the units are British, each inch may be divided either into tenths, or into eighths or into sixteenths. Observations are more readily recorded when the inch is divided into tenths.

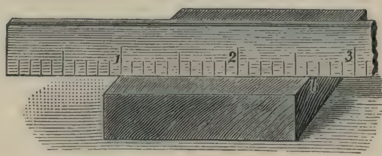


FIG. 1.—Correct way to use a scale.

The following precautions must be observed:

(1) The scale must be held so that the divisions are actually in contact with the line to be measured (Fig. 1). This precaution is necessary in order to avoid errors due to **parallax**, which can be understood by reference to Fig. 2, where the scale reading of the point *a* evidently depends upon the position of the observer's eye.

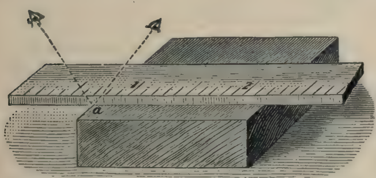


FIG. 2.—Wrong method of using a scale.

(2) The zero end of the scale must not be used, since the end is frequently more or less worn away: some definite division, other than the zero, of the scale should be used (Fig. 1).

(3) Since the position of the point under observation may not coincide with any one division of the scale, it is necessary to estimate fractions of a scale division; thus the scale reading of the point *b* (Fig. 1) is between 2·6 and 2·7, and by regarding each division as divided into 10 equal parts, it is evident that the reading is expressed more accurately by the number 2·63.

EXPT. 1.—**Relation between inches and centimetres.** Measure the distance between the dots AB, BC, and CD in inches and in tenths; measure the distance AD, and see whether this agrees with the sum of the first three measurements.

Make the same measurements in centimetres

Calculate how many centimetres are equivalent to one inch.

A B C D

Tabulate your observations thus:

	In inches (in.).	In centimetres (cm.).
AB	....	....
BC	....	....
CD	....	....
Total.	....	....
AD	....	....

$$\text{Hence, 1 inch} = \frac{\text{length AD, in cm.}}{\text{length AD, in in.}} = \ldots\text{cm.}$$

EXPT. 2.—**Measurement of curved lines.** (i) **By means of dividers.** Draw any curved line in your permanent

note-book. Open out the legs of a pair of dividers until the points are exactly 0·5 cm. apart; make a fine pencil-mark on the curve and place one point of the dividers on the pencil-mark, place the second leg on the curve, raise the first leg and rotate the dividers on the second leg until it is on the curve and beyond the second leg; repeat this process, while counting the number of lengths measured by the dividers, until the end of the curve is reached. Any portion of the curve, less than 5 mm., which remains, must be measured separately by readjusting the dividers.

The length of the curve is given by the product of the distance between the divider points and the number of lengths measured by the dividers. Repeat the measurement with the points of the dividers exactly 1 cm. apart. Note both results, and state which you consider to be the more correct.

(ii)—**By means of cotton thread.** Cut one end of a piece of cotton thread cleanly with scissors, and place the end in contact with a pencil-mark on the curve (Fig. 3). Make the thread coincide as nearly as you can with a small part of the curve, and place the nail of the first finger of your right hand upon it. Now release your left-hand finger and carefully place it at the point where your right-hand finger is held; then, using your right hand, go on to make some more of the thread exactly coincide with another small length of curve. Repeat this until you have completed the whole curve. Measure the length of thread with a millimetre scale.

FIG. 3.—Measurement of a curved line (thread method).

(iii)—**By means of tracing paper** (Fig. 4). Cut a narrow strip of tracing paper, and mark in pencil a central line along it. Place the beginning A of this line over the point where the curved line begins, and stick a pin through the two points. Rotate the tracing paper until as much as possible of the two lines coincide. Transfer the pin to the point where the two lines begin to separate, and again rotate the paper. Proceed in this manner until the end of the curved line is reached. Measure, by means of a scale, the total distance between the extreme points used on the tracing paper. Fig. 4 represents the tracing paper being rotated round the pin at B; and C is the next point to which the pin must be transferred.

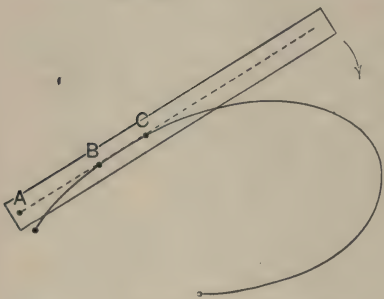


FIG. 4.—Measurement of a curved line (tracing paper method).

EXPT. 3.—**Ratio of the diameter to the circumference of a circle.** Cut out from a sheet of thin cardboard three discs of 4 cm., 6 cm. and 8 cm. radius. Make a pencil-mark near to the edge of each disc; place one of the discs in a vertical position with its pencil-mark touching an observed division of a millimetre scale; roll the disc along the scale until the mark again touches the scale. The difference between the two scale readings gives the length of the **circumference** of the disc. Determine the circumference of the other discs in the same manner.

Tabulate your results thus :

Diameter.	Circumference.	$\dfrac{\text{Circumference.}}{\text{Diameter.}}$
1.	....	....
2.	....	....
3.	....	....

The ratio, in the last column, should be a *constant* quantity : it is usually denoted by the Greek letter π. Hence

length of circumference = $\pi \times$ diameter = $2\pi \times$ radius.

The vernier.—The addition of a vernier to an ordinary scale enables lengths to be measured accurately to a given fraction of the shortest division on the scale used. The simplest vernier scale is one which enables lengths to be read to $\frac{1}{10}$ of a scale division ; and, in this case, the vernier scale is constructed by dividing 9 divisions of the standard scale into 10 equal parts. Fig. 5 represents the method of using such a vernier for measuring the length of an object A ; M is an inch scale divided into tenths, and V is the vernier scale.

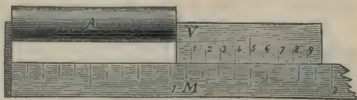


FIG. 5.—Model of a vernier.

The length of A is evidently between 1 inch and 1·1 inch ; and the vernier scale enables the amount by which the length exceeds 1 inch to be measured accurately. It will be noticed that the divisions of the two scales *coincide at one point only*, which is approximately, the 4th of the vernier scale. Since 1 division of scale V is equal to $\frac{9}{10}$ of one division of scale M, *i.e.* to 0·09 inch, the 3rd division of V is 0·01 inch in advance of the division 1·3 on scale M, and

the 2nd division of V is 0·02 inch in advance of the division 1·2 on scale M,

the 1st division of V is 0·03 inch in advance of the division 1·1 on scale M,

the zero division of V is 0·04 inch in advance of the division 1·0 on scale M.

The last quantity, 0·04 inch, is the fraction of a division which had to be measured. Hence, the length of A is 1·04 inches.

An alternative method of reasoning is as follows:

The length of A + the length of 4 vernier divisions = 1·4 inches. Hence, the length of $A = 1{\cdot}4 - (4 \times 0{\cdot}09)$
$= 1{\cdot}04$ inches.

It is evident that the following rule may be adopted in using such a vernier scale: *Note where divisions on the two scales coincide; the number attached to the division of the vernier scale which coincides gives the numeral in the second place of decimals.*

It may be desired to use a vernier reading to a smaller fraction than 1/10 of a scale division; in this case 19 scale divisions may be divided into 20 equal parts, giving a vernier-scale reading to 1/20 of a scale division. Generally speaking, in order to read to $1/n$ of a scale division, $(n-1)$ divisions must be divided into n parts.

EXPT. 4.—**Construction of a vernier model.** Draw accurately on thick white paper an inch scale, about 4 inches long, divided into tenths of inches, and a vernier scale in which 0·9 inch is divided into 10 equal parts. Cut out these scales and mount them carefully on a wooden model as shown in Fig. 5. Use this model for the measurement of several suitable objects.

Slide calipers.—With the aid of slide calipers it is possible to measure with considerable accuracy (1) the linear dimensions of solid bodies, (2) the internal diameters of tubes, and (3) the depth of hollow vessels. Fig. 6 represents one of the several types of slide calipers. The upper jaws are used for measurements of internal dimensions, and the lower jaws for external dimensions. The left-hand jaws are fixed to the steel bar, and the right-hand jaws to the slider, which can be fixed in any position by means of a screw. An inch scale, divided into sixteenths, is engraved on one edge of the bar, and a millimetre scale

on the other edge. Two vernier scales are engraved on the bevelled surfaces of the slider.

When the slider is moved so that the jaws are in contact, the zero line of each vernier should coincide with the zero of the scales. In Fig. 6 the jaws are separated to the extent of exactly one inch. On examining the inch scale

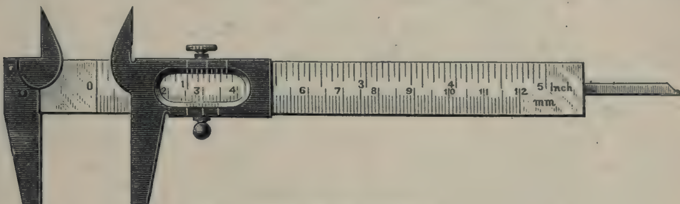


FIG. 6.—Slide calipers.

and its vernier, it will be seen that seven scale-divisions are divided into eight equal parts on the vernier; hence this vernier reads to $(\frac{1}{8} \times \frac{1}{16}) = \frac{1}{128}$ inch. In the same manner, it will be seen that the other vernier reads to 0·1 mm.

Depths of hollow vessels are measured by means of the narrow metal strip which slides in a groove at the back of the scale bar, and is attached to the slider. This is pushed outwards until its extreme end touches the bottom of the cavity while the end of the scale bar rests on the edge of the cavity; the reading of the vernier then gives the depth of the cavity.

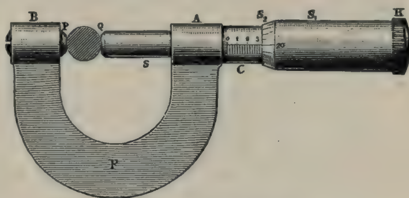


FIG. 7.—Screw-gauge.

The screw-gauge.—The screw-gauge (Fig. 7) furnishes a very accurate means for measuring the dimensions of small objects. It consists of a fixed frame F, attached to which is a hollow cylinder C. A screw thread is cut on the inside surface of C. The shaft S is the continuation of a screw which travels along the threads cut within C; and a sleeve S_1 is attached to the head H of the screw. The edge S_2

of the sleeve is divided into a definite number, usually 50 or 100, of equal parts. The end of the shaft S is a truly planed surface Q, and a similar surface P is obtained on the end of a fixed screw carried by the other limb B of the fixed frame. This screw is adjusted, once for all, so that, when the edge of S_2 coincides with the zero division of the scale on C and when the zero division of the scale S_2 coincides with the base line of the scale on C, the two plane faces P and Q are in contact.

Before taking a measurement, it is necessary to observe whether the scale on C is divided into millimetres or into tenths of an inch. The *pitch* of the screw S—*i.e.* the distance through which Q advances or recedes by one complete rotation of H—must then be determined. This is obtained by observing whether one division, or only *half* a division, of the scale on C is uncovered when H is rotated backwards by one complete rotation. Finally, the scale-value of one division of the scale S_2 is required.

As a general rule, the pitch of the screw S is 0·5 mm., and S_2 is divided into 50 equal parts; hence 1 division of

$$S_2 = \tfrac{1}{50} \times 0{\cdot}5 = 0{\cdot}01 \text{ mm.}$$

The object to be measured is placed between the faces P and Q, and the milled head H is rotated until the object is *lightly* gripped between the faces. The readings of the scales on C and on the sleeve at S_2 enable the size of the object to be measured.

EXPT. 5.—**Measurements with the screw-gauge.** Select pieces of bare copper wire, of different thickness, the *number* of each on the *Standard Wire Gauge* (S.W.G.) being known. Measure the diameter of each piece, taking measurements at not less than three different points of each piece. Record each reading; and if for any specimen the readings differ, calculate the mean diameter. Refer to a printed table of the S.W.G. dimensions,[1] and note whether your observations agree with these.

[1] See Table, p. 235.

Measure in the same manner the thickness of a microscope cover-glass, or of a thin piece of plate glass. Reserve the specimen measured so as to verify the readings by means of the spherometer.

Measure the thickness of fifty sheets of your textbook; and calculate the thickness of one sheet.

The spherometer.—The principle of this instrument closely resembles that of the micrometer screw-gauge. The instrument consists of a tripod, the legs of which are of equal

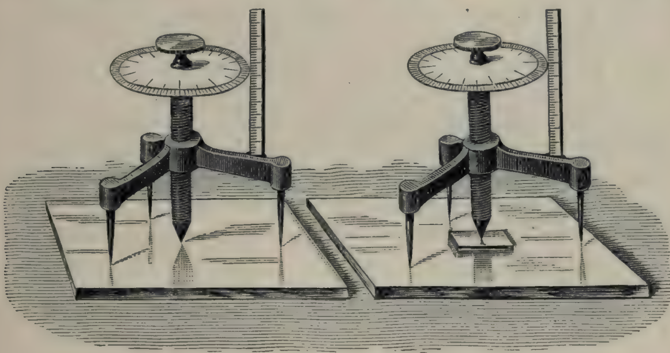


FIG. 8.—Use of a spherometer.

length and are adjusted relatively to each other so that the three points occupy the corners of an equilateral triangle. A fine screw, which works through the centre of the tripod, terminates above in a milled head and a large circular disc, the edge of which is divided into 100 equal parts; the point of this screw is equidistant from each of the legs. A vertical scale, usually divided into millimetres, is fixed to one arm of the tripod, and with its divisions close to the edge of the disc.

Before using the instrument, it is necessary to determine the *pitch* of the screw: this may be equal to 1 mm. or to 0·5 mm. This is done by reading the position of the disc's edge on the vertical scale, and then rotating the disc

through an observed number of complete turns; the difference in the scale-reading divided by the number of turns gives the pitch of the screw. If the pitch of the screw is 0·5 mm., and if the disc is divided into 100 equal parts, then 1 division of the disc is equal to 0·005 mm.

In measuring the thickness of an object, the following procedure is adopted: Place the instrument on a truly horizontal surface, *e.g.* a sheet of plate glass, and rotate the screw downwards until its point *just* touches the surface (Fig. 8); this is determined most accurately by placing the thumb and first finger against opposite sides of one leg of the tripod, and endeavouring to make the instrument rotate round the centre leg; if the latter projects downwards too far the instrument readily rotates, but if contact is not complete there is an unmistakable sense of resistance to rotation. Having made this adjustment, take the reading of the scale and disc. Now raise the screw considerably, place the object underneath the screw point, and rotate the screw downwards until contact is just made again. The difference between the two sets of readings gives the thickness of the object.

EXPT. 6.—**Measurement with the spherometer.** Measure the thickness of the glass used in Expt. 5. Also, for exercise in using the instrument, measure the dimensions of some small metal cubes or similar objects.

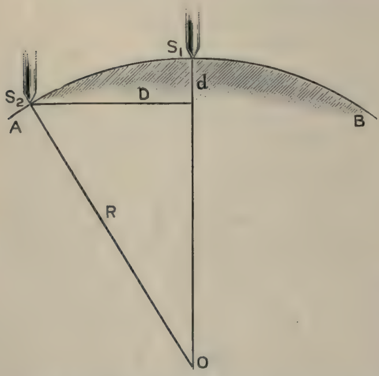


FIG. 9.—Measurement of curvature, by the spherometer.

The application of the spherometer to the measurement of the curvature of a spherical surface explains the origin of the name of the instrument. Let AB (Fig. 9) represent

a portion of a spherical surface with its centre at O, and let S_1 and S_2 represent respectively the central leg and one of the outer legs of a spherometer. The length D is the horizontal distance between S_1 and S_2, and d is the vertical distance through which the screw must be raised so that it just touches the curved surface. If R is the radius of curvature, it is evident that

$$R^2 = D^2 + (R - d)^2,$$

or

$$R = (D^2 - d^2)/2d.$$

EXPT. 7.—**Measurement of curvature with the spherometer.** Use, as a spherical surface for measurement, either a lens or a mirror not less than two inches in diameter. Take the reading of the spherometer when adjusted on a sheet of plate-glass; and measure accurately the distance between the point of the screw and each of the legs. In an accurate instrument these distances should be the same, and the measurement represents the length D of Fig. 9. If the distances differ, take the mean of the three readings as the value of D.

Place the spherometer on the curved surface, adjust the screw for exact contact, and take the reading. The difference between the two readings is the length d of Fig. 9. Use the above equation to calculate the value of R. Note the number or description of the curved surface so that the result may be verified subsequently by an optical method (p. 154).

ANGLE.

Measurement of angles.—The general plan adopted in measuring angles is to divide a circle into 360 equal parts, and to call each part a **degree** (1°). The magnitude of a given angle can be measured, or a stated angle can be

described, by means of a **protractor**, two forms of which are shown in Fig. 10. The simplest form is a metal, or celluloid, semi-circle divided into degrees; another common

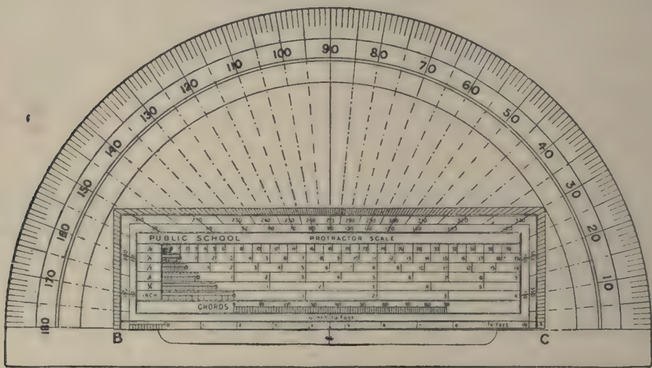


FIG. 10.—Semi-circular and rectangular forms of protractor, used for the measurement of angles.

form is oblong in shape and made of boxwood. The marks upon the edge of the latter are obtained from the corresponding divisions on a semi-circular protractor in the manner represented in Fig. 10.

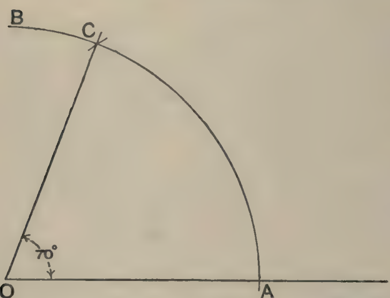


FIG. 11.—Angle described by the scale of chords.

A boxwood protractor usually has a **scale of chords** stamped upon its surface; this can be seen in Fig. 10. Suppose that an angle of 70° is to be described: Draw any line OA (Fig. 11). With a pair of pencil compasses measure off on the scale of chords the distance from the zero of the scale to the line numbered 60. With centre at O, describe an arc BA of a circle with radius equal to this distance. Alter the compasses to the distance from

the zero of the scale to the number on the scale representing the required angle (in this case, 70). With centre A describe an arc cutting the arc BA at the point C. Join OC. Then COA is the angle required.

EXPT. 8.—**Use of a protractor.** (i) Describe, by means of the scale of chords, angles of 40°, 55°, and 80°. Verify the angles by means of a circular protractor.

(ii) Draw several angles of different magnitude. Measure them (*a*) by means of the scale of chords, and (*b*) by means of a circular protractor.

TIME.

The unit of time.—In physical measurements the unit of time is the **mean solar second**, which is the 86,400th part of the **mean solar day.** This latter is the average time required by the earth to make one complete rotation on its axis relatively to the sun considered as a fixed point of reference.

The rate of a clock is regulated usually by means of a pendulum, which swings to and fro at a rate depending almost entirely upon its length, and is independent of the amplitude of its swing provided that the amplitude is small. A **seconds pendulum** is one of which the length is such that consecutive passages across the position of rest are separated by a time-interval of exactly one mean solar second.

A **simple pendulum** may be defined as a **heavy particle suspended by a weightless thread.** An approximation to this ideal is obtained by suspending a small metal sphere by a very thin thread. In the arrangement represented in Fig. 12, stout cotton is threaded along the axis of a cork, which serves as the carrier for the pendulum. The *bob* consists of a truly turned solid brass sphere through which a very narrow hole is bored along a diameter; this

hole is bored out to larger size for a short distance from one end.

In fitting up the pendulum the cotton is threaded through the bob, held with the wide end of the bore downwards; the cotton is knotted sufficiently for the knot to pass just within the wide end of the bore. The upper end of the cotton is held firmly in a deep slit cut vertically in a cork; and the length of the pendulum is varied readily by pulling more or less of the cotton through the cork.

FIG. 12.—A simple pendulum.

The diagram indicates how the length of the pendulum may be measured by means of a metre scale and a wooden cube. The true length of the pendulum is approximately the distance from the point of support to the *centre* of the bob; and this length is best obtained by measuring the distance to the bottom of the bob and subtracting from this the radius of the bob. In the following experiment a cheap stop-watch is desirable.

EXPT. 9.—**Length and rate of swing of a pendulum.** Support immediately behind the thread of the pendulum a piece of cardboard on which a vertical pencil line is drawn. Sit down in front of the pendulum, and, *using one eye only*, adjust the position of the cardboard so that the thread exactly covers the pencil line. Keep the eye in this same position during the following observation.

Set the pendulum swinging through a small angle not exceeding 15°. A single swing from side to side is termed usually a **vibration**, and a swing-swang, or

complete movement to and fro, is an **oscillation**. Note the time indicated by the stop-watch, and start the watch just as the thread is passing in front of the pencil line.

Count the number of subsequent passages of the thread until at least 50 vibrations have been completed, and stop the watch at an instant when the pendulum is passing the pencil line. Read the watch, and calculate the time interval between two consecutive passages of the pendulum in front of the pencil line.

Repeat the experiment; and if this result differs from the first determination by more than 0·01 second, take a third observation. Measure the length of the pendulum. Alter the length of the pendulum, and determine the time of vibration again in the same manner. Repeat this for different lengths, varying from 20 cm. to 120 cm.

Tabulate your results thus:

Length.	Time of vibration.	$\sqrt{\text{Length}}$.	$\dfrac{\sqrt{\text{Length}}}{\text{Time of vibration}}$.

Plot on squared paper the *length* and the *time of vibration*, taking the latter as **ordinates**, that is, values reading upwards on the squared paper, while lengths are read horizontally. Similarly plot the $\sqrt{\textit{length}}$ and the *time of vibration*, or *length* and *time*^{2}. From the curves obtained deduce the relationship between the time of vibration and the length of the pendulum.

The following readings were obtained with a simple pendulum constructed in the manner described above:

Length.	Time of vibration.	Length.	Time of vibration.
20 cm.	0·45 sec.	88 cm.	0·94 sec.
30 „	0·55 „	95 „	0·98 „
42 „	0·65 „	102 „	1·01 „
55 „	0·74 „	115 „	1·07 „
70 „	0·835 „	130 „	1·14 „

Fig. 13 represents how these readings may be plotted on squared paper. Two **axes**, OX and OY, are drawn at right

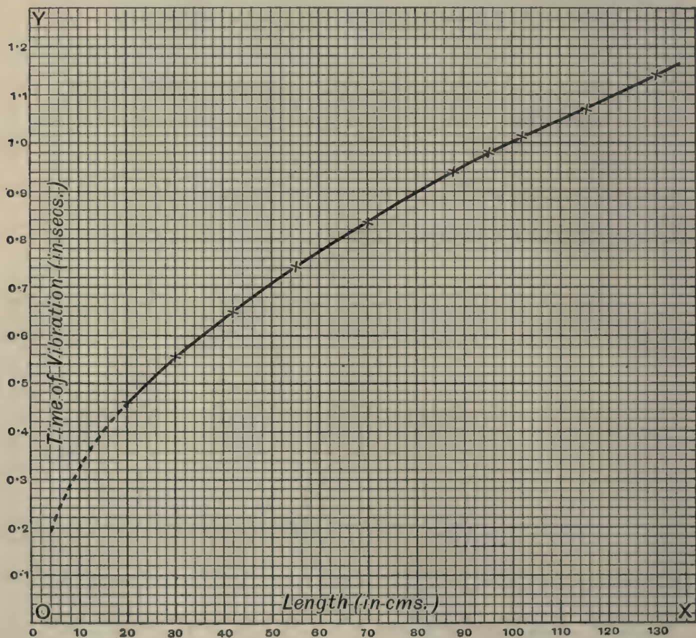


FIG. 13.—Graphical representation of the relation between the length and time of vibration of a simple pendulum.

angles to each other, with the point O near to the left-hand bottom corner of the paper. The line OX is called the axis of **abscissae**, and OY the axis of **ordinates**. For the purpose of the present experiment, the horizontal scale along OX is taken to represent the *length* of the pendulum, and the scale along OY is taken to represent the time of vibration.

In the diagram, each division of OX represents 2 cm., and each division of OY represents 0·02 sec. In general, the value attached to each scale division should be chosen so that the length of the two scales utilised for plotting the observations are nearly equal, and so that the lengths are as great as the paper will allow.

Bearing in mind the simplicity of the apparatus, the liable error of observation, and the possibility of irregularity in the squared paper, it cannot be expected that all the plotted readings may coincide with the curve (or straight line) which represents a theoretically accurate result. Having plotted the points which indicate the observations taken, a line should be drawn which, as nearly as possible, passes through each point.

CHAPTER II.

THE MEASUREMENT OF AREA.

Units of area.—The unit of area used in experimental work is either the **square inch** or the **square centimetre**, according to whether the British or the metric system is used. Since 1 inch = 2·54 cm.,

$$\textbf{1 sq. in.} = (2{\cdot}54)^2 = \textbf{6·45 sq. cm.}$$

This relationship can be verified in several of the experiments with squared paper described in this chapter.

Errors of observation.—The calculation of an area, from measurement of its dimensions, always involves the multiplication of two observed lengths. If the measurements are taken by means of an ordinary millimetre scale, it is possible to estimate accurately a fraction of the smallest scale division only to about one-fifth of a division, *i.e.* to about 0·2 millimetre; and when the total length is small, this liable error is considerable. Since **an area is the product of two lengths**, both of which are liable to error, the liable error of their *product* is much greater; in fact, if each length has a liable error of 1%, the liable error of their product will be 2%.

Thus, suppose a 1 inch square be drawn, and two adjacent sides measured with a millimetre scale. Each side may be judged to be 2·54 cm. long; but the third significant figure is *estimated* only, and the length *might* be as low as 2·53 cm. or as high as 2·55 cm. The liable error, therefore, is at least 1 in 250, or 0·4%. If the lower figure is correct, the area would be 6·401 sq. cm.; whereas the area calculated from the estimated length 2·54 cm. would be

6·452 sq. cm. The liable error in the calculation is therefore about 0·8%. Thus in the figures 6·452 sq. cm., the digit 2 is quite meaningless; the digit 5 is very untrustworthy, and therefore should be omitted. Hence, when the lengths are as small as those quoted, *any figures beyond the third significant digit are to be omitted in the statement of the result of an experiment.* When the lengths are greater the liable percentage error is necessarily much smaller; for this reason it is always an advantage, in such experiments as those which follow, to draw the diagrams on as large a scale as possible.

USE OF SQUARED PAPER.

EXPT. 10.—**Ratio of the square inch to the square centimetre.** Use paper ruled in tenth of inch squares. Draw in pencil, in *any* position on the paper, a 6 cm. square (Fig. 14). Count the number of small squares enclosed, marking off as shown the unbroken sets of 25 squares. *Where the outline of the area cuts through a square, count it as a whole square if the portion enclosed is greater than one-half, but disregard it if it is less than one-half.* Record the observations thus:

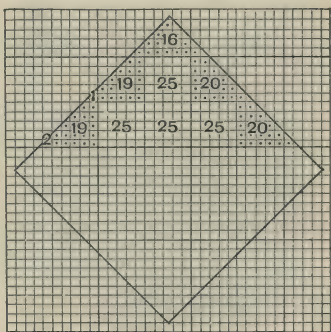


FIG. 14.—Area, by counting squares.

Number of small squares = ?

Area = ?/100 sq. in.

Area = 6 × 6 = 36 sq. cm.

Hence, $$1 \text{ sq. in.} = \frac{36}{?/100} = \ldots \text{ sq. cm.}$$

EXPT. 11.—**Area of a triangle.** Draw on squared paper any irregular triangle ABC (Fig. 15), *e.g.* with

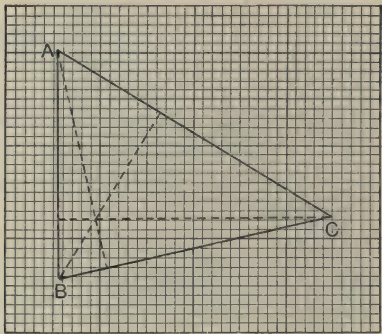


FIG. 15.—Area of a triangle.

sides 2·5 in., 3 in. and 3·5 in. long. Find its area by counting squares. Compare this result with that obtained from the formula

$$\textbf{area} = \tfrac{1}{2}(\textbf{base} \times \textbf{height}).$$

Take in succession (i) BC as base, (ii) AB as base, and (iii) AC as base.

EXPT. 12.—**Area of a parallelogram.** By the same method as in Expt. 11, verify that

$$\textbf{area} = (\textbf{base} \times \textbf{height}).$$

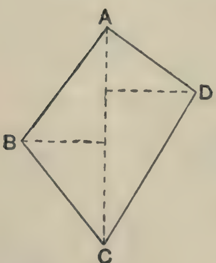


FIG. 16.—Area of any four-sided figure.

EXPT. 13.—**Area of any four-sided figure.** Draw any four-sided figure ABCD (Fig. 16). Divide it into two triangles by joining either pair of opposite corners, *e.g.* AC, and calculate from its dimensions the area of each triangle. Verify that the sum of these areas is equal to the area found by counting squares.

EXPT. 14.—**Area of a circle.** Draw a circle of any suitable radius, *e.g.* 1·5 in. On any radius of this circle construct a square. Count (i) the small squares within the circle, and (ii) those within the square described on the radius. Verify that the ratio (area of circle)/(area of square on radius) = π. (See Expt. 3.) This demonstrates experimentally the familiar formula

$$\textbf{area of a circle} = \pi r^2,$$

where r is the radius of the circle.

EXPT. 15.—**Area of an ellipse.** Draw an ellipse on squared paper by means of two pins and a cotton loop

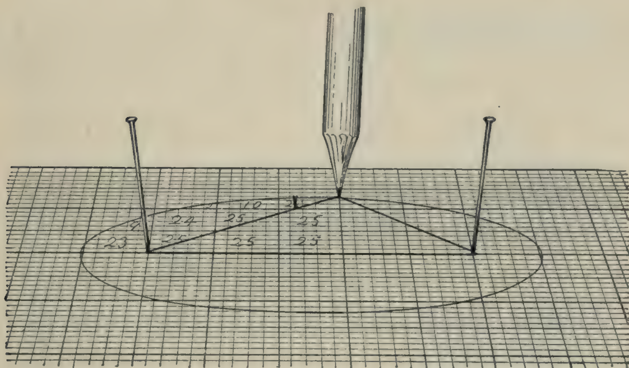


FIG. 17.—Area of an ellipse.

(Fig. 17). Find its area by counting squares and verify that

$$\textbf{area of an ellipse} = \pi ab,$$

where a and b are the semi-axes of the ellipse.

Irregular areas.—The use of squared paper for measuring irregular areas may be exemplified by its application in

finding, by means of a trustworthy map, the area enclosed within any given boundary. Thus, in Fig. 18, the outline of the island of Ceylon has been traced in pencil on tracing paper; this paper was then placed face downwards on squared paper, and the outline re-traversed by a hard point, thus transferring the outline to the squared paper. By counting squares, the area (in sq. in.) is obtained; and the scale on the map serves to calculate the number of square miles equivalent to one square inch. The following observations apply to Fig. 18:

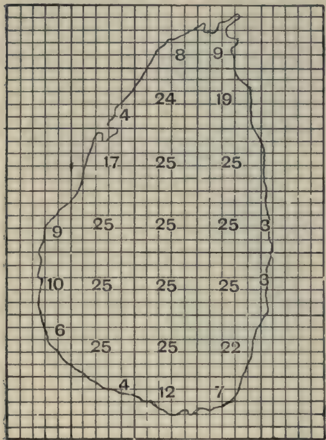


FIG. 18.—A geographical area.

Ruling of the paper.—1 small square $=0{\cdot}01$ sq. in.

Number of small squares.—407.

Area enclosed.—$4{\cdot}07$ sq. in.

Scale of map.—150 geographical miles $=1{\cdot}88$ in.,
or 1 inch $=79{\cdot}7$ miles,
or 1 sq in. $=6287$ sq. miles.

Hence, *Area of Ceylon* $=4{\cdot}07\times 6287=25{,}600$ sq. miles.

[In Whitaker's Almanack, the area of Ceylon is stated to be 25,000 sq. miles.]

EXPT. 16.—By the method described above, find the area of two other countries or provinces.

ENGINEERS' METHODS.

Simpson's rule.—Areas of irregular outline are frequently computed by **Simpson's rule**, the method of which is as follows: Divide the area into an *even* number of strips

by drawing across it a series of equidistant ordinates. The number of such ordinates will be *odd.* Measure the length of each ordinate. Add together the first and the last ordinate, *twice* the sum of the other *odd* ordinates, and *four* times the sum of the *even* ordinates; multiply this sum by one-third of the distance between consecutive ordinates. This product is the area of the space enclosed.

Suppose that the area is divided into eight strips, each h inch wide. There will be nine ordinates; if the lengths of these are represented by $y_1, y_2, y_3, \dots y_9$, then

$$\textbf{area} = \frac{h}{3}\{y_1 + y_9 + 2(y_3 + y_5 + y_7) + 4(y_2 + y_4 + y_6 + y_8)\} \text{ sq. in.}$$

EXAMPLE.—Fig. 19 represents an area of irregular

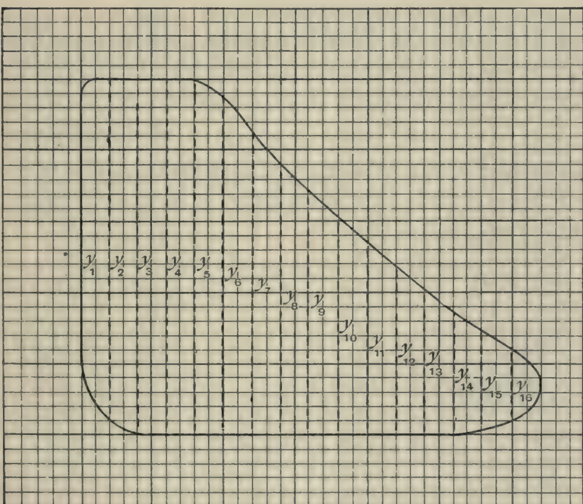


FIG. 19.—Determination of area by Simpson's rule.

outline, somewhat similar to an *indicator-diagram* of a steam engine. The area is divided into sixteen strips;

and, of the seventeen ordinates, the 17th is equal to zero.

MEASUREMENTS:

$$h = 0{\cdot}2 \text{ in.}$$
$$y_1 = 1{\cdot}90 \text{ in.}$$
$$y_{17} = 0.$$
$$2(y_3 + y_5 + \ldots + y_{15}) = 23{\cdot}76 \text{ in.}$$
$$4(y_2 + y_4 + \ldots + y_{16}) = 52{\cdot}88 \text{ in.}$$

Hence,

$$\text{Area} = \frac{0{\cdot}2}{3}\{1{\cdot}90 + 23{\cdot}76 + 52{\cdot}88\} = 5{\cdot}24 \text{ sq. in.}$$

This result was verified by counting squares. The number of small squares was 525; the area, therefore, is 5·25 sq. in.

EXPT. 17.—(i) The following figures give the depth (in feet) of the water in a river at a number of points along a straight line at right angles to the banks, the distance (x feet) of each point from one of the banks being given. Plot the readings to a suitable scale on squared paper and find, by Simpson's rule, the area of cross-section of the river.

x (distance)	0	10	20	30	40	50	60
y (depth)	0	7·8	12·1	15·2	17	18·1	18·1
x (distance)	70	80	90	100	110	120	
y (depth)	17·4	15·5	13·0	10	7·0	2·2	

(ii) Find the area of a horizontal half-section of a ship at the water-level, of which the curved form is defined by the following equidistant ordinates (in feet) spaced 12 ft. apart: 0·1, 5·1, 7·17, 8·75, 10·1, 9·17, 8·05, 6·4, 0·1.

The planimeter.—Goodman's **hatchet planimeter** [1] (Fig. 20), which is the simplest available type of planimeter, is capable of giving results accurate to within 3 % when used with care. It consists of a rigid bar of metal bent twice at

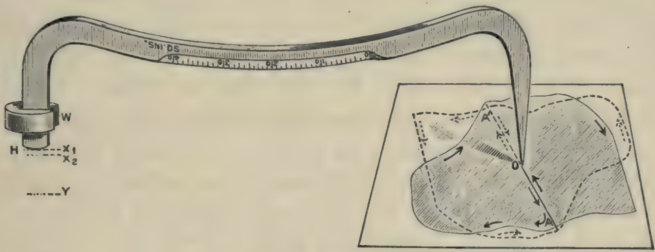


FIG. 20.—The 'hatchet' planimeter.

right angles; one end terminates in a point, and the other in a hatchet-shaped knife-edge H, which is loaded with a detachable weight W. On the lower edge of the bar a scale is engraved which enables the area to be read off directly in square inches.

EXPT. 18.—**Measurement of area by means of the hatchet planimeter.** Fasten a large sheet of smooth paper on a *horizontal* surface, and place on this sheet the paper on which the area to be measured is drawn. Estimate approximately the centre of area O, and through O draw any straight line OA from O to the boundary of the area. Set the planimeter with its point on O; hold the instrument vertically, and adjust it so that its length is perpendicular to the line OA. Apply slight pressure over the knife-edge so that a shallow depression X_1 is marked on the paper. *Always holding the instrument vertically*, make the point trace out the line OA, then round the boundary of the area *in a clockwise direction*

[1] The instrument is manufactured by Messrs. Reynolds & Branson, Ltd., Commercial Street, Leeds. (Price 15s.)

to A, and finally back to O. The knife-edge will now occupy some new position Y; press down the edge so as to mark this position.

Now, using the point as a centre and without moving the knife-edge, rotate the paper through 180° so that the line OA now occupies the position OA′ and the area is as represented by the dotted outline. Make the point traverse the line OA′ round the area *in an anti-clockwise direction* and back to the point O; again press the knife-edge, which may (or may not) occupy its initial position. If the final position is X_2, estimate by eye the mid-point between X_1 and X_2, and with the scale measure the distance between this mid-point and the mark Y, always using the zero of the scale. The scale-reading gives the area in square inches. Verify the result either by means of Simpson's rule or by squared paper.

In order to become competent in handling the instrument, the student is recommended to practise previously with regular areas, such as rectangles and circles.

CHAPTER III.

THE MEASUREMENT OF VOLUME.

Units of volume.—The volume of any bulk of a substance is expressed in terms of either the **cubic centimetre** (*c.c.*) or the **cubic inch** (*c. in.*): these units are represented by a cube each edge of which is either 1 cm. long or 1 in. long.

The consideration of a simple type of regular solid, such as the rectangular block shown in Fig. 21, shows that a volume is expressed numerically by the product of three lengths. If the length OL, the breadth OB and the depth OD are respectively equal to 6 units, 4 units and 3 units of length, the entire solid may be divided into three layers each containing (4×6) unit cubes. The total volume therefore is $(4 \times 6 \times 3) = 72$ units of volume.

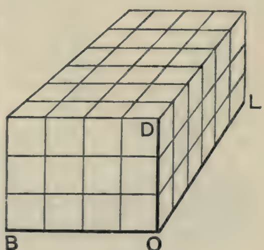


FIG. 21.

Hence, the procedure for calculating the volume of a rectangular solid is to measure the lengths of any three concurrent edges, such as OL, OB and OD, and to multiply these together. It must be remembered that whatever is the liable error in the measurement of each of these three lengths, the liable error in the product will be three times as great.

Fig. 21 will explain the following equalities:

$$1 \text{ cubic yard (c. yd.)} = 3^3 = 27 \text{ c. ft.}$$
$$1 \text{ c. ft.} = 12^3 = 1728 \text{ c. in.}$$
$$1 \text{ c. in.} = (2{\cdot}54)^3 = 16{\cdot}39 \text{ c.c.}$$
$$1 \text{ cubic decimetre (or } \textbf{litre}) = 10^3 = 1000 \text{ c.c.}$$

USE OF GRADUATED VESSELS.

Vessels used as metric measures of capacity.—A number of vessels used in the measurement of capacity are shown in Fig. 22. The **flask** marked 500 c.c. has a mark upon its neck, and when filled up to this mark the number shows how many cubic centimetres of liquid are in it. The tall jar (or **cylinder**) has a mark at every 10 cubic centimetres up to 1000 c.c.; the number of cubic centimetres of a liquid may thus be found by pouring the liquid into the jar and reading the scale division which is on the same level with the surface. Since the volume indicated by each scale division varies according to the diameter of the cylinder, it is necessary, before using any cylinder, to determine by means of the numbers attached to the scale the capacity represented by consecutive scale divisions.

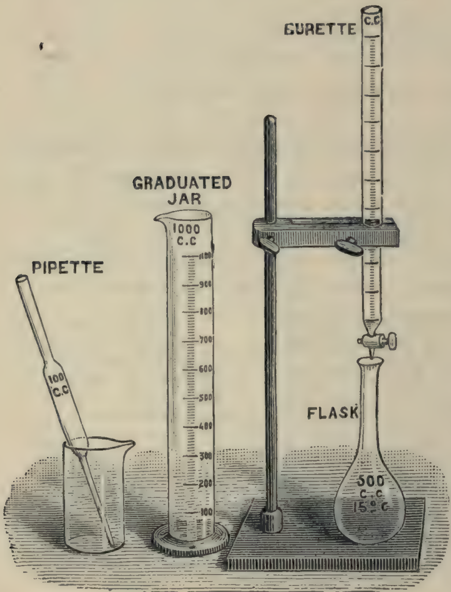


FIG. 22.—Graduated vessels for measurements of volume.

The graduated tube is a **burette,** used for measuring out exact quantities of liquid. At the bottom is a tap or clip for allowing liquid to flow out of the burette. Supposing that the burette is filled to the mark 10 c.c., and that 35 c.c. of liquid are required from it, the tap would be opened gradually, and when the liquid had fallen to the mark

45 c.c. it would be closed quickly. Burettes are graduated always from the top *downwards.* When a burette is first filled with the liquid, the tap or clip should be opened and the liquid allowed to run out until all air has been expelled from the exit tube: the instrument is then ready for use.

The narrow tube supported in a beaker is a **pipette,** by means of which small quantities of liquid may be conveyed from one vessel to another.

Fig. 23 indicates the correct method of reading the position of the liquid surface (or **meniscus**) in a measuring vessel. The surface is curved downwards (except when mercury is used), and it appears to have an upper and lower margin: the scale reading of the middle point of the lower margin should be obtained always. This is observed most satisfactorily if the meniscus is illuminated by holding a piece of white paper horizontally close to the glass vessel and two or three inches below the meniscus. The eye must be on a level with the meniscus: the sloping dotted lines show how incorrect readings are obtained if this precaution is not taken.

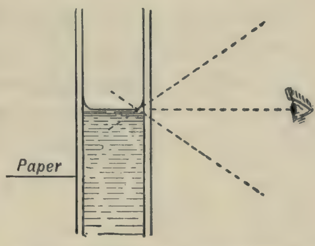


FIG. 23.—Observing the height of a liquid surface in a tube.

Methods of **calibrating** (*i.e.* determining the errors, if any, in the scales or marks etched on the vessels) measuring vessels are described in Chapter IV.

Measuring vessels provide a convenient means of determining the volume of a solid by observing the volume of water displaced by the solid when immersed in the water.

EXPT. 19.—**Volumes by displacement and calculation.** Find, by the method described, the volume of the following regular solids—(i) a cube, (ii) a cylinder, (iii) a pyramid, (iv) a cone, and (v) a sphere. Select a measuring cylinder into which the solid will just pass

without friction, and pour in a considerable volume of water. Read the position of the meniscus.

Attach a very thin wire to the solid, and lower it into the cylinder until it is immersed *completely*; again read the position of the meniscus. The difference between the two readings indicates the volume of the solid.

If the material of the solid floats in water, use, instead of the wire, a needle fixed to the end of a wooden penholder, and thus push the solid until it is completely immersed.

Verify the result, in each case, by calculating the volume from measurements of the dimensions. Thus,

(i) volume of **rectangular solid** = **length** × **breadth** × **depth**;

(ii) „ „ **cylinder** = **base** × **height** = $\pi r^2 h$;

(iii) „ „ **pyramid** (with square base)

$$= \tfrac{1}{3} \times \textbf{base} \times \textbf{height} = \frac{a^2 h}{3};$$

(iv) „ „ **cone** = „ „ $= \dfrac{\pi r^2 h}{3}$;

(v) „ „ **sphere** $= \tfrac{1}{3} \times$ **surface** × **radius** $= \dfrac{4\pi r^3}{3}$.

EXPT. 20.—**Volume of solids, by overflow apparatus.** The apparatus of Fig. 24 can be made either from an inverted bell jar, or from a bottle of which the bottom has been cut off. A narrow glass tube passes through the cork; near the top of the tube a small hole is blown (by a blowpipe) or bored, through which the overflowing water can pass. The lower end of this tube is ground or filed to an acute angle: this prevents any water from accumulating at that end.

Support a burette below the open end of the tube. Completely fill the jar with water, so that some overflows into the burette. Read the meniscus in the

burette. Gently immerse the solid in the water, and when no more overflows, again read the burette. The difference in the readings gives the volume of the solid.

This apparatus is useful when a measuring cylinder of suitable diameter is not available.

FIG. 24.—Expt. 20.

EXPT. 21.—**Average volume of lead shot.** About half-fill a burette with water, and read the meniscus. Select a given number (*e.g.* 20 or 30) of the shot, and introduce them into the burette—which should be held in a slanting position. Again read the meniscus. From the volume of the displaced water calculate the average volume of one shot.

If the student is familiar with the use of a balance, the following method of finding the volume of small quantities of a solid substance is the best to adopt. The *weight* of the displaced water can be determined by means of a balance with far greater accuracy than its *volume* can be determined by means of a measuring vessel. The method requires also previous knowledge of the fact that, at ordinary temperatures, one cubic centimetre of water weighs 1 gram (approximately).

EXPT. 22.—**Volume of a solid by weighing the displaced water.** Place the solid fragments, of which the volume is required, in a porcelain crucible, and fill a 'specific gravity' bottle with cold water. Place both of these on the pan of a balance (Fig. 25 (i)) and weigh them. Remove the bottle and crucible from the pan, and transfer the fragments to the bottle. Shake the contents of the

bottle, to remove any air bubbles, completely fill the bottle with cold water, insert the stopper and clean the outside of the bottle. Replace the crucible and

(i) (ii)

FIG. 25.—Expt. 22.

bottle on the pan, and again weigh (Fig. 25 (ii)). The difference in the two weighings represents the weight of water displaced by the solid; from this the volume of the solid can be directly stated.

When the solid is soluble in water it is necessary to use some other liquid, *e.g.* alcohol, instead of water. It is necessary then to determine previously the weight of 1 c.c. of the liquid used.

CHAPTER IV.

MEASUREMENT OF MASS AND OF DENSITY.

Distinction between mass and weight.—It is desirable to make a distinction between the terms **mass** and **weight**. The mass depends solely upon the quantity of material in the object, and it is quite independent of its position relatively to the earth. The weight of the same mass is the force of attraction which the earth exerts upon it, and this force depends upon the distance of the mass from the centre of the earth. Hence, the mass is constant, while the weight is variable.

A **spring-balance** measures the earth's pull upon the mass attached to it, and therefore records its weight. On the other hand, in the use of an ordinary **beam-balance** two masses are adjusted in quantity until their weights are equal. The readings, therefore, obtained by means of a beam-balance will be the same in all localities, while those obtained by means of a spring-balance will vary.

Although it is incorrect to do so, it has become customary to use the expression *weighing* whichever instrument is used.

The balance.—Fig. 26 represents a simple form of beam-balance. A brass beam AB is supported at its centre by a knife-edge of hard steel which, when the balance is in use, rests on a true surface of similar steel. The hooks to which the pans are attached are provided with a **V**-shaped depression of hard steel which, when the balance is swinging, rests upon knife edges on the upper edge of the beam. A

pointer F attached to the beam indicates, by means of the ivory scale, the swinging of the beam. When not in use,

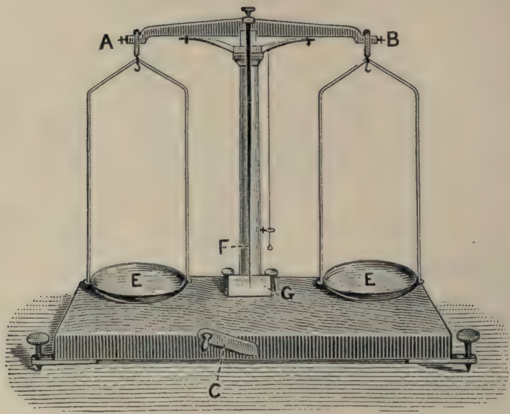


FIG. 26.—A beam-balance.

and when weights are being changed, the beam is lowered on to rigid supports by turning the handle C.

Set of metric weights.—Sets of metric weights are marked usually in grams and milligrams, and a complete set will include the following:

(i) (Brass weights), 100 gm., 50 gm., 20 gm., 20 gm., 10 gm., 5 gm., 2 gm., 2 gm., 1 gm.

(ii) (Aluminium weights),

500 mgm. (=0·5 gm.), 50 mgm. (=0·05 gm.),
200 mgm. (=0·2 gm.), 20 mgm. (=0·02 gm.),
200 mgm. (=0·2 gm.), 20 mgm. (=0·02 gm.),
100 mgm. (=0·1 gm.), 10 mgm. (=0.01 gm.).

By means of this set any weight which is a multiple of 10 mgm. and does not exceed 211 gm. may be obtained. The box contains a pair of **forceps**, which must be used always when weights are removed from, or replaced in, the box.

The weights are used in the following manner: the object to be weighed is placed in the left pan, and such weights as are estimated as sufficient to counterbalance the object are placed in the right pan.

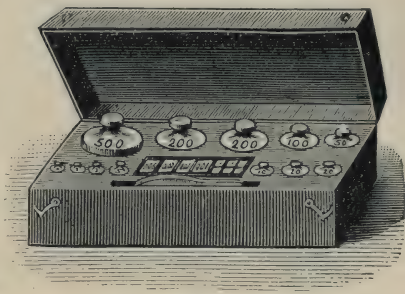


FIG. 27.—Metric weights.

Suppose that 20+10 gm. are used, and that, on *slightly* releasing the beam, the pointer moves towards the object: these weights are evidently too great. The 5 gm. weight is substituted for the 10 gm. weight; if this is now too small, a 2 gm. weight is added; and, if this is still too small, the second 2 gm. weight is added. The subsequent steps are as follows:

20+5+2+2, too great.
20+5+2+1, too small.
20+5+2+1+0·5, too small.
20+5+2+1+0·5+0·2, too great.
20+5+2+1+0·5+0·1, too great.
20+5+2+1+0·5+0·05, approximately correct.

The weight is thus found to be 28·55 gm.

Special precautions in weighing.— 1. See, by means of the plummet, that the balance is level.

2. See that the stirrups are not displaced; see also that the pans are dry and clean.

3. Lower the arrestment to see whether the pointer swings equally on both sides of the middle point of the scale. If necessary, adjust the balance by means of the screw-nuts at either end of the beam.

4. Do not stop the swinging of the balance with a jerk, but stop it gently when the pointer is nearly at its central position.

5. Place the body to be weighed on the left-hand, and the weights on the right-hand, pan.

6. Lower the arrestment before adding or removing any weight.

7. Manipulate the arrestment with the left hand, and convey weights with the right hand. On no account touch weights with the hand, but always use the forceps.

8. Do not weigh a body when hot: the heat causes air currents, which affect the weighing.

9. Close the balance case when observing the swinging of the pointer, and keep the case closed when the balance is not in use.

10. Always replace each weight in its proper compartment in the box.

Sensibility of a balance.—In determining the sufficiency of the weights on the balance pan it is unnecessary to wait until the beam ceases to swing; it is sufficient to observe the extreme positions on the scale to which the pointer travels, and from these the final position of rest can be calculated. The scale divisions should be numbered, as shown in Fig. 28. Suppose that consecutive turning points are 8 and 14; the true position of rest will not be far from $\frac{8+14}{2}=11$.

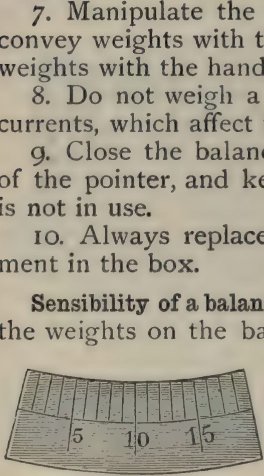


FIG. 28.—Scale divisions of a balance.

Strictly speaking, we ought to allow for the fact that the extent of swing is gradually becoming less, owing to friction and air resistance. We should take, therefore, two consecutive readings on *one* side and the intervening reading on the other side. Thus, suppose the readings are 8 (left), 14 (right), 8·4 left; the average of the 1st and 3rd gives the reading on the left which would have been obtained if the vigour of the swing at the moment of taking the right turning point remained unaltered. Hence, the true turning point on the left is $\frac{8+8{\cdot}4}{2}=8{\cdot}2$; and the true position of rest is $\frac{8{\cdot}2+14}{2}=11{\cdot}1$.

The **sensibility** of a balance may be defined as *the change in the position of rest of the pointer due to a given change of weights*; it is expressed usually as the change of weights necessary to alter the position of rest through one scale division. The sensibility of the simple balances generally used in elementary experiments varies from 2 to 4 milligrams per scale division; it varies slightly, according to the load on the balance, becoming less with heavy loads; and it is usually a maximum with a load of 20-30 grams.

EXAMPLE.

I. With 20 gm. in each pan.

	Left.	*Right.*
	7·8	...
	...	14·0
	8·0	...
Mean	7·9	

$$\text{Resting Point} = \frac{7{\cdot}9 + 14}{2} = 10{\cdot}95.$$

II. With 0·01 gm. added to right pan.

Left.	*Right.*	
...	9·4	
5·0	...	
...	9·2	
	9·3	mean.

$$\text{Resting Point} = \frac{5{\cdot}0 + 9{\cdot}3}{2} = 7{\cdot}15.$$

Hence, change in *Resting Point*, due to 0·01 gm.,

$$= 10{\cdot}95 - 7{\cdot}15 = 3{\cdot}8 \text{ divisions.}$$

Therefore, **sensibility** $= \frac{10}{3{\cdot}8} = 2{\cdot}6$ mgm. per scale division.

EXPT. 23.—**Sensibility with and without load.** Find the sensibility of a balance, with (i) no load, (ii) a load of 20 grams in each pan, (iii) a load of 50 grams in each pan.

When recording the swings, close the front of the balance case, and allow the pointer to swing to and fro several times before taking the readings.

Weighing by vibrations.—In weighing an object it is not necessary to modify the known weights until the resting point is the same as that obtained when the balance is unloaded, providing that the sensibility of the balance is determined previously.

EXAMPLE.

Sensibility of balance = 3 mgm. per scale div.

Resting point, with no load, = 11·5.

Resting point, with object on left, and 21·55 gm. on right, = 9.

The weight, 21·55 gm., is evidently *too great* by an amount equivalent to 11·5 − 9 = 2·5 div.

But,	1 div. is equivalent to	3	mgm.,
or	2·5 div. are „ „	7·5	„

Hence, true weight = 21·55 − 0·0075 = 21·5425 gm.

If the sensibility is not known with sufficient certainty, the following procedure may be adopted:

EXAMPLE. Resting point, with no load, = 11·5

Resting point, with	21·55 gm. on left,	= 9.
„ „	21·54 „ „	= 12·4.

Hence 10 mgm. are equivalent to 3·4 scale div.

The weight 21·54 gm. is evidently *too small* by an amount equivalent to (12·4 − 11·5) = 0·9 scale div.

But 0·9 scale div. is equivalent to

$$\left(10 \times \frac{0{\cdot}9}{3{\cdot}4}\right) = 2{\cdot}6 \text{ mgm.};$$

$$\therefore \text{ true weight} = 21{\cdot}54 + 0{\cdot}0026 = 21{\cdot}5426 \text{ gm.}$$

EXPT. 24.—**Vibration method.** Find, by the method of vibrations, the weight, in grams, of a 1 oz. weight. Repeat the experiment, using a ½ oz. weight and a 2 oz. weight.

CALIBRATION.

It is found frequently that graduated vessels, such as the pipette and burette, are not sufficiently correct for use in accurate experimental work. The process of determining the error, if any, in the graduations is termed **calibration.** The following experiments illustrate the principle of an approximate method of calibrating the above types of measuring vessel, and provide also exercises in weighing.

EXPT. 25.—**Calibration of a pipette.** Use distilled water which has been standing in the room for several hours. Note its temperature. Carefully clean the inside of a 10 c.c. pipette.[1] Clean and dry a small wide-necked flask. Cover the neck with a small watch-glass, and weigh the flask with the cover. Fill the pipette with the distilled water, and adjust the meniscus to the graduation mark. Transfer the water to the flask by holding the point of the pipette against the inside of the neck; when empty, blow down it once. Cover the flask with the watch-glass, and again weigh it.

The increase in weight expressed in grams gives in c.c. the capacity of the pipette; when the temperature of the water is not higher than 15° C., the assumption that 1 gm. of water occupies 1 c.c. introduces an error of less than 1 in 1000.

If a considerable error is found in the graduation, fix a narrow strip of gummed paper along the upper stem of the pipette, and find by trial the position which the meniscus must occupy in order that the pipette may deliver exactly 10 c.c. Mark this position permanently by means of a scratch made with a file.

EXPT. 26.—**Calibration of a burette.** Clean the inside of the burette, and fix it vertically in a stand. Fill it with distilled water at the temperature of the room. Run some of the water out at the tap or jet, so as to remove all air bubbles. Carefully run more water out until the meniscus coincides with the zero of the scale; and remove any drop adhering to the end of the jet. Weigh a flask together with a small watch-glass to serve as a cover. Run water from the burette into the flask until the meniscus is at the 10 c.c. mark; and touch the

[1] The best method of removing grease, and other matter, from a glass vessel is to rinse it thoroughly with strong sulphuric acid with a little potassium bichromate added. If possible, leave this liquid in the vessel for an hour; then rinse several times with tap water, and finally with distilled water.

inside of the flask with the end of the jet. Replace the watch-glass, and again weigh the flask. Run water into the flask until the meniscus is at the 20 c.c. mark, and again weigh. Continue this process of weighing successive quantities, of 10 c.c. each, until the 50 c.c. mark is reached.

The total increase in weight (in gm.) of the flask at each weighing gives the approximate true volume delivered between the zero and each of the graduated marks used.

Plot on squared paper the total apparent volume of water taken from the burette, and the total increase in weight of the flask.

EXPT. 27.—**Calibration of a capillary tube.** Select a length of thermometer-tubing with circular bore (1-2 mm. diameter) and about 1 metre long. If necessary, chemically clean and dry the inside of the tube. Make a file-mark about 1 cm. from one end. Attach to one end a long piece of narrow rubber tubing; and, by this means, suck up into the tube a column (about 10 cm. long) of *pure* mercury. Note the temperature of the mercury.

Place the tube horizontally on an accurately divided metre-scale, so that the file-mark coincides with the zero of the scale. By tilting the tube adjust the position of the mercury thread so that the centre of the thread is approximately over the 5 cm. mark of the scale, and carefully measure the length of the thread. Move the thread along the tube until the centre is over the 15 cm. mark and again measure the length.

Repeat this measurement with the centre of the thread over the 25 cm., 35 cm., etc. divisions of the scale, until the end of the tube is reached. Weigh accurately a porcelain crucible. Transfer the mercury thread into the crucible and again weigh.

Having given that the density (see p. 44) of mercury at the temperature of the room is approximately 13·56 gm. per c.c., calculate the average cross-section of the tube for each position of the thread in the tube.

Plot on squared paper the position of the centre of the thread and the average cross-section in each position.

Example of Calibration of a Capillary Tube.

Temp. of mercury, 16° C.

Weight of crucible, 7·484 gm.

" " + mercury, 9·246 gm.

" mercury alone, 1·762 gm.

Cross-section of tube = Weight ÷ (Length × Density)

= 1·762 ÷ (Length × 13·56)

= 0·130 ÷ Length.

Position of centre of thread.	Length of thread.	Cross-section.
5	10·00 cm.	0·0130 sq. cm.
15	9·95 "	0·01306 "
25	9·77 "	0·0132 "
35	9·90 "	0·01313 "
45	10·06 "	0·01292 "
55	10·02 "	0·01297 "
65	9·92 "	0·0131 "
75	9·92 "	0·0131 "
85	9·90 "	0·01313 "

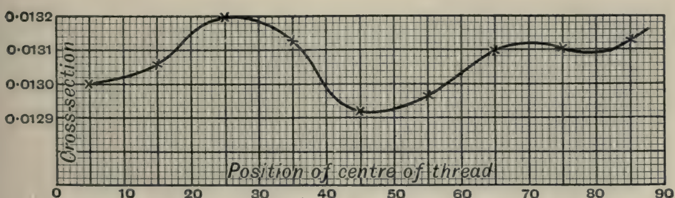


Fig. 29.—Calibration curve of a capillary tube.

Weighing with an ill-adjusted Balance.

Expt. 28.—**Borda's substitution method.** Place some shot or sand in a box or dish on the left-hand pan, using sufficient to have a greater weight than that of the object to be weighed. Place known weights (w_1) on the right-hand pan until the balance is counterpoised, and obtain the resting point. Remove the weights, place the object on the right-hand pan, and add weights (w_2) until the balance is counterpoised again. Obtain the resting point, and calculate the correction to w_2 necessary to make the two resting points coincide. The weight of the object is then equal to $(w_1 - w_2)$ grams.

Expt. 29.—**Gauss's method, by double weighing.** This method is particularly applicable when the arms of the balance are of unequal length: weigh the object (i) when placed on the left-hand pan, and (ii) when placed on the right-hand pan.

If the arms are of unequal length the apparent weight in the two cases will differ. The true weight is equal to the geometrical mean of the two apparent weights. Thus, if X is the unknown weight, L and R the lengths of the left and right arms respectively of the balance, and W_1 and W_2 the apparent weights, then

$$\text{(i)}\quad X \times L = W_1 \times R\,; \quad \text{or} \quad L/R = W_1/X,$$

$$\text{(ii)}\quad X \times R = W_2 \times L\,; \quad \text{or} \quad L/R = X/W_2.$$

Hence $$W_1/X = X/W_2,$$

or $$X^2 = W_1 W_2,$$

or $$X = \sqrt{W_1 W_2}.$$

The Spring Balance.

Expt. 30.—**The spring balance; and Hooke's Law.** Clamp rigidly the upper end of a spiral of thin steel

wire (Fig. 30) the lower end of which is twisted so as to supply an attachment for a small pan to carry weights, and the free end projects horizontally beyond the side of the spiral. Clamp a half-metre scale vertically so that the pointer of the spiral travels over the scale divisions. Note the scale-reading. Add weights, 10 gm. at a time, to the pan, and note the scale-reading after each addition. Continue the observations until the spring is nearly twice its original length. Now remove the weights, step by step, and note the readings as before.

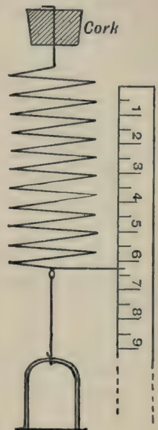


FIG. 30.—A spring-balance.

Plot the observations on squared paper, taking *weights* as abscissae and the *scale-readings* as ordinates. From the *graph* thus obtained, state your inference as to the relationship between the weight applied and the resulting elongation of the spring; and state whether any permanent elongation is evident.

The results of Expt. 30 show that, in the case of a spring, **the amount of elongation is proportional to the load.** This relationship holds good generally for the stretching of wires, rods, or similar test-pieces, provided that the limits of elasticity are not exceeded; that is, they should return to the original length when the load is removed. Using the word **strain** to signify the deformation or distortion produced by a load, and **stress** for the internal forces, which are equal to the distorting force, tending to bring the body back to its original form, the relationship is expressed in the statement: **The strain is proportional to the stress** (**Hooke's Law**). This law applies to any deformation of an elastic body, such as bending or twisting, as well as to stretching.

Density.

The **density** of a substance is defined as **the mass of unit volume**, and is expressed usually either in *grams per c.c.* or in *pounds per c. ft.*

In expressing the density of any material it is essential to state the units in which the measurements are taken. Thus, the mass of 1 c.c. of water, at 4° C., is 1 gram, and that of 1 cubic foot of water is 62·5 lb. Hence, the density of water is **1 gm. per c.c.** (in the metric system) or **62·5 lb. per c. ft.** (in the British system). It is *not* sufficient to say that the density of water is either *1*, or that it is *62·5*; each statement is incomplete, and it is essential to state the units which have been used.

On the other hand, when it is desired to express the **relative density** (or **specific gravity**) of a material—*i.e.* the number of times that the material is denser, or less dense, than a standard material—a simple numerical figure is correct. Thus, the *density* of copper is 8·8 *gm. per c.c.*, but the *relative density* of copper is 8·8.

Expt. 31.—**Density of regular solids.** Select a cube, a cylinder, a cone, and a sphere, of different materials. Calculate the volume of each solid from its dimensions, and also weigh each of them. Calculate the density of each material. Tabulate your results thus:

Material and shape.	Dimensions.	Volume.	Mass.	Density.
Brass (cylinder)	Radius = Length =	$(\pi r^2 \times l) =$		

Expt. 32.—**Irregular solids.** Select a glass stopper, and carefully examine it to see that it is free from internal cavities. Weigh it, and determine its volume by displacement of water. Calculate its density.

Carry out similar observations with other irregular solids.

EXPT. 33.—**Liquids.** Weigh a clean, dry beaker. Partly fill a pipette (20 c.c. or 25 c.c. capacity) with the liquid; rinse it round the inside of the pipette, and throw the liquid away. Completely fill the pipette with the liquid up to the mark on the stem, and transfer the contents to the beaker. Weigh the beaker and the liquid contained in it. From these weighings, and from the known volume of the pipette, calculate the density of the liquid.

CHAPTER V.

ATMOSPHERIC PRESSURE. BOYLE'S LAW.

EXPT. 34.—**The density of air.** Fit a one-holed india-rubber stopper into a fairly large glass flask, and fit into the stopper a short glass tube with rubber tubing and clip (Fig. 31). Put a little water in the flask; open the clip; and boil the water. After boiling for some minutes, close the clip and place the flask on one side to cool. When the flask is cool, weigh it. Then open the clip; air will be heard to rush into the flask, and as it does so the balance will show an increase of weight. Carefully re-weigh the flask. The increase in weight is equal to the weight of air within the flask.

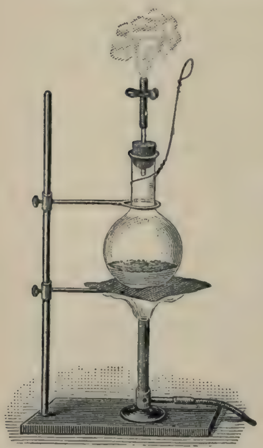


FIG. 31.—Apparatus for finding the density of air.

Measure the water in the flask by means of a measuring cylinder; *fill* the flask with water up to the position occupied by the bottom of the stopper, and measure its volume. The difference of these volumes gives the volume of the air which entered the flask. From these results calculate the *weight of one litre of air.*

In Expt. 34 it is important to remember that the air contained in the flask is saturated with water-vapour; hence the space must be regarded as being occupied partly by air and partly by water-vapour. Since water-vapour is less dense than dry air, the density calculated from the experiment will be slightly less than would be obtained if dry air had been weighed. The difference amounts only to about 0·7 %.

The error due to the presence of water-vapour may be avoided by using a glass vessel similar to that represented

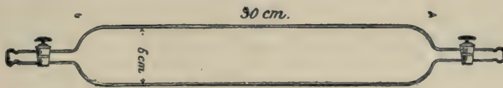


FIG. 32.—Glass vessel, for Expt. 35.

in Fig. 32. In this case the air is removed by means of an air pump, and the procedure is as follows:

EXPT. 35.—**Alternative method.** Support the tube (Fig. 32) horizontally on a tripod. Connect one end to a *drying-tube* containing granulated calcium chloride, and the other end to a filter pump. Open both taps and draw a *slow* current of air through the apparatus; at the same time heat the tube slightly and uniformly. Continue this for some time, then turn off the tap at the end near the pump. Leave the other tap open until the tube has cooled to the temperature of the room. Read the barometer and the temperature of the room. When both taps are closed, disconnect the apparatus, suspend the tube by fine wire from the hook of a balance and weigh it accurately. Remove the tube, connect it to an air pump by means of thick-walled rubber tubing and withdraw as much air as possible. Again weigh the tube. The difference in weight represents the weight of air removed.

To determine the volume of the air removed, transfer the tube to a trough of water, *which must be at the same temperature as the room.* Open *one* of the taps, and hold the tube in a slanting position so that the level of the water inside and outside the tube is the same. Turn off the tap, remove the tube, carefully dry the outside, and again weigh it.

Since one gram of water occupies one c.c. approximately, the increase in weight represents numerically the volume of water admitted, and therefore of the air previously removed. Calculate the weight of one litre of air.

EXAMPLE. Temp. of room, 17° C.
Barometer, 759 mm.
Sensibility of loaded balance, 1 scale division = 13·7 mgm.

Weight.	Zero of balance.	Weight, corrected to zero, 10·19.
i. 142·20	10·19	
ii. 141·98	8·68	141·953
iii. 344·35	11·02	344·361

Weight of air removed,

$$142{\cdot}20 - 141{\cdot}953 = 0{\cdot}247 \text{ gm.}$$

Volume of air removed,

$$344{\cdot}361 - 141{\cdot}953 = 202{\cdot}41 \text{ c.c.}$$

Hence, weight of 1 litre of air (at 17° C. and 759 mm.)

$$= \frac{0{\cdot}247}{0{\cdot}20241} = 1{\cdot}220 \text{ gm.}$$

EXPT. 36.—**Fitting up a simple mercury barometer.** Procure a thick glass tube about 36 inches long and closed at one end. The tube must be quite clean and

dry. Fill the tube with *dry*, *clean* mercury,[1] leaving a small air bubble at the top. Close the tube with the thumb, and slope the tube downwards so that the bubble of air travels along the whole length of the tube; slope the tube upwards, so that the bubble returns to the open end. Thus all small air bubbles are removed from the sides of the tube. Fill up the tube with mercury, place your thumb over the open end; invert the tube; place the open end in a cup of mercury and take away your thumb.

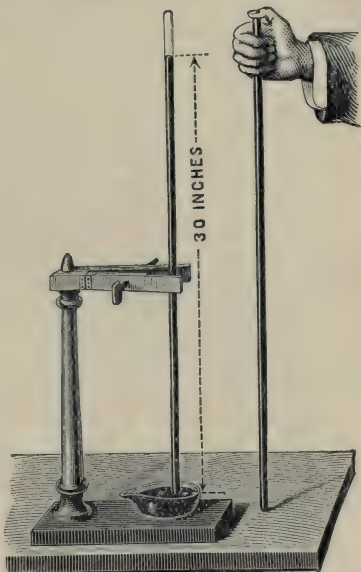


FIG. 33.—A simple barometer.

Test whether the space above the mercury column is a good vacuum by tilting the tube away from the vertical position until the mercury touches the top of the tube. Notice whether the mercury strikes the glass with a dull 'metallic click,' or whether it 'rebounds' several times as though it struck an elastic cushion, thus indicating that air is present.

[1] Dirty mercury can be cleaned satisfactorily by allowing it to stand for several days in contact with concentrated sulphuric acid. A long narrow *separating-funnel* is the most convenient appliance to use, as mercury may be drawn off as required through the glass tap at the bottom of the funnel. It may be desirable finally to filter the mercury through a cone of glazed paper, in the apex of which a pin-hole opening has been made; the paper cone should be supported in an ordinary funnel.

Measure the height of the mercury column above the surface of the mercury in the cup. Assuming that the density of mercury is about 13·56 gm. per c.c., calculate the pressure of the air on each sq. cm. of the mercury in the cup.

If a standard barometer is available, verify the accuracy of your determination of the barometer reading.

BOYLE'S LAW.

Fig. 34 represents a simple form of apparatus for observing the effect of change of pressure on the volume of any gas.

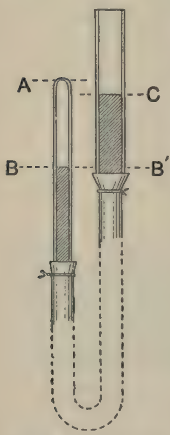


FIG. 34.—Apparatus for Boyle's Law.

The glass tube on the left is about 30 cm. long and 5 mm. internal diameter, and it is sealed at the upper end. The tube on the right is somewhat shorter and wider, and is open at the upper end. The tubes are joined by a considerable length of *thick-walled* rubber tubing[1]; the ends of the rubber are tied on firmly by wire.

In setting up the apparatus, proceed as follows:—Straighten out the apparatus, hold it in a slightly inclined position with the open end upwards, and pour in clean mercury until it is about two-thirds full. Erect the tubes and, when the mercury has settled down, observe the quantity of air which remains in the closed tube; if this is not a convenient quantity, dismount the tubes, close the open end with the thumb and transfer air through the mercury—whichever direction is desired—until the desired quantity of air is contained in the closed tube. The quantity should be such that

[1] Thick-walled tubing is usually vulcanised; and, when new, its surface is coated with finely divided sulphur. This should be removed by rinsing the inside of the tubing with carbon bisulphide; otherwise the mercury will be rendered dirty, thus preventing accurate observations.

when the tubes are erected so that the two mercury surfaces are at the same level, they occupy about the mid-position of each tube.

If the air is damp the moisture present introduces errors in the observations. When it is desired to avoid this liable error, the closed tube must be replaced by one which terminates at the top with a glass tap. The apparatus is then filled by attaching to the tap a drying tube containing calcium chloride. The tap is opened and the right-hand tube is raised until the mercury completely fills the other tube; on lowering the right-hand tube dry air is drawn into the other tube and is enclosed by turning off the tap.

When the closed tube is graduated, the volume of the enclosed air is observed directly by reading the position of the mercury surface on the scale; but if the tube is not graduated, the *length* of the air column may be taken as numerically equal to its volume, providing that the tube is of uniform bore. If, in Fig. 34, the vertical heights above the bench of the levels A and B are measured with a metre scale, the difference in the readings may be taken as a measure of the volume of the enclosed air.

When the tubes are adjusted so that the two mercury surfaces are at the same level, the pressures on the two surfaces must be equal. Hence the pressure on the enclosed air must be equal to the pressure of the atmosphere to which the exposed surface is subjected, and this is observed by reading the height of the barometer at the time of the experiment. In order to vary the pressure acting on the enclosed air, the open tube is raised or lowered. In Fig. 34 the open tube is represented as having been raised; the pressures in the two tubes *at the level* BB′ are the same, *and the pressure at* B′ *is equal to the atmospheric pressure at* C *added to the height of the mercury column* CB′; hence the pressure acting on the enclosed air is $(H+h)$ cm. of mercury, where H is the barometer reading, and h is the difference of level CB′. When the open tube is lowered so that the surface C is *below* the surface B, the difference of level h must be *subtracted* from the barometer reading.

EXPT. 37.—**Effect of change of pressure on the volume of a gas.** Take a series of readings of the *pressure* acting on the enclosed air, in an apparatus similar to Fig. 34, and of the *volume* of the air. Every precaution must be taken to *keep the temperature of the enclosed air constant* during the period of the experiment. The necessary data are obtained by measuring the vertical distance above the bench of the levels A, B and C.

Record the observations thus:

Height of Barometer, (H).	Difference of Level of Mercury, (h).	Total Pressure on the Air, $P=(H \pm h)$.	Volume of Air, (V).	Total Pressure × Volume, ($P \times V$).
Cm.	Cm.			
....	(i)	....	....	....
	(ii)	....	....	....
	(etc.)			

It will be found that, within the limits of experimental error, when there is no change of temperature the product of the volume of any given mass of gas and the pressure to which it is subjected is always the same. In other words, **the volume of a given mass of gas varies inversely as its pressure.**

Plot the above readings on squared paper, taking pressures as abscissae and volumes as ordinates.

EXPT. 38.—**Simple method for Boyle's Law.** Obtain a length of thermometer tubing (Fig. 35), about 75 cm. long and 1 mm. bore. Seal it at one end and expand the open end somewhat. Clamp the tube in a vertical position, with the closed end below, by the side of a metre scale, and connect a small funnel to the top by means of a short piece of rubber tubing. Pour a little pure, clean mercury into the funnel and induce

it to run down the bore of the tube by inserting a thin, clean, steel wire. In this way any desired volume of air can be enclosed.

The length of the column of enclosed air may be taken to represent its *volume* (V). If H = the height of the barometer, and h = the length of the mercury thread (both expressed in the same units), then the total pressure on the enclosed air = $(H + h)$.

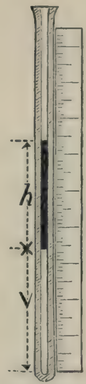


FIG. 35.—Simple experiment on Boyle's Law.

Introduce more mercury in the same manner, and in this way alter the values of V and h. The volume of the air under the pressure of the atmosphere alone can be observed by laying the glass tube flat on the table; and the volume under pressures less than that of the atmosphere can be observed by inverting the tube with its open end downwards.

Perform several experiments and record the results in the following way:

Volume (V).	Pressure $(H \pm h)$.	Volume × Pressure.

CHAPTER VI.

PRESSURE IN LIQUIDS. PRINCIPLE OF ARCHIMEDES. RELATIVE DENSITY.

Pressure at a point within a liquid.—The pressure at any point within a liquid is defined as the average pressure acting on an imaginary horizontal square centimetre passing through the point; and it is expressed either in grams weight or in dynes.[1]

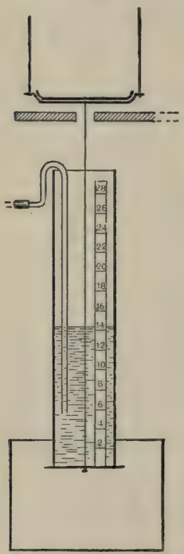


FIG. 36.—Apparatus for measuring pressure within a liquid.

The pressure, due to the liquid only, is numerically equal to the weight of a column of the liquid having a cross-section of one sq. cm. and height equal to the vertical depth of the point below the surface of the liquid.

The pressure on any horizontal surface of area A sq. cm., immersed to a depth h cm. below the surface of a liquid having a density of d gm. per c.c., is expressed by the product $(A \times h \times d)$ grams. This assumes that no external pressure is acting on the surface of the liquid.

This principle can be verified by the apparatus shown in Fig. 36. A wide glass tube 40 cm. long and about 4 cm. diameter has its lower end made perfectly true by grinding it on a sheet of plate glass with a mixture of fine emery powder

[1] The dyne is the *absolute* unit of force; it is equal to the weight of one gram divided by the acceleration due to gravity expressed in metric units. Hence the dyne = $\frac{1}{981}$ gram approximately.

and turpentine. A paper millimetre scale is fixed on the outside so as to measure distances from the lower end. A circular glass cover plate is supported horizontally by means of a fine wire passing through a small hole bored in the centre of the plate. The tube is fixed firmly in a vertical position so that the fine wire, after passing through a hole bored in the base-board of a balance, can be attached to the metal carrier of the balance pan.

When the cover plate and suspension have been counterpoised, the addition of a further weight to the other pan of the balance presses the cover plate firmly against the end of the tube. A *slow* stream of water can now be led into the wide tube, by means of a narrow glass tube reaching nearly to the bottom of the apparatus; and the water will be retained until the total pressure on the cover plate is equal to the additional weight put into the other pan of the balance. Casual leakage from the tube is minimised by previously rubbing a thin film of oil over the upper surface of the cover plate.

EXPT. 39.—**Pressure within a liquid.** Having provided the necessary parts for fitting up the apparatus of Fig. 36, measure by means of calipers the internal diameter of the lower end of the wide tube, and calculate its cross-section. Fit up the apparatus, and place below it a large trough to collect the water which flows out of the tube.

Counterpoise the cover plate, and adjust the position of the wide tube so that with the beam of the balance raised the pointer is at zero when the cover plate is in contact with the tube. Add a weight of 200 grams to the other pan of the balance, and allow water to run slowly into the tube. Carefully watch the surface of the water column and notice its scale-reading at the moment when the cover plate suddenly descends.

Readjust the cover plate and repeat the observation

at least three times. Take the average height of the water column and calculate the total pressure on the cover plate. Repeat this experiment, using weights of 300, 400 and 500 grams.

Enter your observations thus:

Diameter of tube = 4·72 cm.

Cross-section „ $= \pi \times \overline{2 \cdot 36}|^2 = 17 \cdot 50$ sq. cm.

Additional weight.	Height of water column.	Average height, *h*.	Weight of water column = (A × *h* × *d*).
(i) 300 gm.	16·6 cm. 16·8 „ 17·1 „	16·83	294·5 gm.
(ii) 400 gm.	23·0 cm. 22·8 „ 23·6 „ 24·1 „	23·40	409·5 gm.

The principle of a floating body: the upthrust. At any point below the surface of a liquid the pressure must be the same *in all directions*: the downward pressure must be neutralised by an exactly equal upward pressure. Similarly the pressures in all other directions must neutralise each other: otherwise, the liquid in the region of the point would be continually moving in the direction of the greater force. Hence, when a solid is floating in any liquid *the downward pressure* (*i.e.* the weight) *of the solid must be neutralised by the upward pressure of the liquid on the immersed surface of the floating body.*

In the simple case of a cylindrical solid (Fig. 37) floating vertically in a liquid, the weight of the solid is neutralised by the upthrust of the liquid on the horizontal immersed end; and this is equal to the weight of the column of liquid displaced by the cylinder. Hence it would be anticipated that

weight of floating solid = weight of displaced liquid.

The apparatus shown in Fig. 37 consists of a piece of

thin-walled glass tubing, about 25 cm. long and 2 cm. diameter. The lower end is closed with a well-fitting slice of cork which has been soaked in melted paraffin wax. Inside the tube and near to the top is pasted a paper scale, made from squared paper (divided into millimetres), and numbered so as to measure exactly distances from the bottom of the tube. In order that the tube shall float vertically in water, small lead shot are poured into the tube until it floats as required. The shot may be fixed into the tube by adding small fragments of sealing wax, which are melted afterwards by heating the tube from the outside. The top of the tube is closed with another slice of cork.

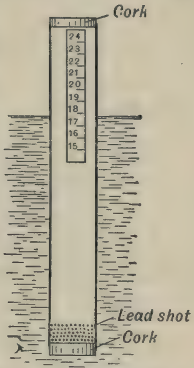


FIG. 37.—Model of a hydrometer.

EXPT. 40.—**Principle of a floating body.** Weigh the completed tube; measure its diameter, and calculate its cross-section. Float it in a glass cylinder containing water, read the depth of immersion and calculate the volume of the immersed part of the tube. Finally, knowing the density of the liquid, calculate the weight of the displaced liquid. Repeat the experiment, using other liquids, *e.g.* brine, dilute methylated spirit, etc.

Enter your results thus:

Weight of floating body =
Cross-section of tube =

Liquid used.	Length immersed.	Volume immersed.	Weight of the displaced liquid.
(i)	....	....	....
(ii)	....	....	....
(iii)	....	....	....

Deduce from the results the relationship between the weight of the floating body and that of the displaced liquid.

The principle of Archimedes.—This principle may be stated thus: **When a solid is immersed in a fluid (liquid or gas) the apparent loss of weight is equal to the weight of an equal volume of the fluid.**

EXPT. 41.—**Principle of Archimedes.** Measure the dimensions of a metal cube, and calculate its volume. Suspend the cube, by means of cotton thread, from the hook of a balance, and at such a height that the cube may afterwards hang freely in a beaker of liquid supported on a bridge placed over the pan. Use as little cotton as possible: a single strand with a loop at the top should suffice. Weigh the suspended cube. Now place a beaker of cold water on the bridge so that the cube is immersed totally, taking care that it does not touch the sides of the beaker, and again weigh the cube.

Repeat the above experiment several times, using regular solids of different form and different liquids of known density.

Tabulate your results in the following manner, and deduce your own conclusions as to the relationship between the apparent loss of weight of the solid and the weight of an equal volume of the liquid used.

Liquid, and its density (d).	Dimensions of solid.	Volume of solid.	Weight of equal volume of liquid.	Apparent loss of weight.

The method of this experiment is the same as that which has been used to determine the magnitude of the *gram* weight, which is the unit of mass in the C.G.S.

system. A cube or cylinder of glass or quartz was measured with extreme accuracy, and its apparent loss of weight observed when suspended in water at 4° C. The ratio *loss of weight/volume* is equal to the weight of 1 c.c. of water at 4° C.; and this, by definition, is the true gram weight

The reaction to the upthrust.—Since the liquid in which a solid is immersed exerts an *upward* vertical force, called the *upthrust*, there must be an equal and opposite reaction. This reaction acts on the base of the containing vessel; and if the vessel and the contained liquid are weighed, there will be found a corresponding apparent *increase* in this weight, and equal in amount to the apparent loss of weight of the immersed solid.

This apparent increase in the weight of the vessel and liquid would be anticipated by remembering that the immersion of the solid causes an increase in the *depth* of the liquid, and therefore an increased pressure on the base of the vessel.

EXPT. 42.—**Reaction to the upthrust.** From the hook of a balance suspend by means of a thin wire a suitable solid: it is convenient to use a glass bulb (Fig. 38), about 2 cm. diameter, containing a little mercury, which has been sealed off, and the end bent to form a hook.

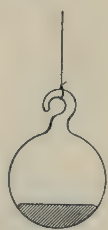


FIG. 38.

Weigh the bulb (i) when hanging in air, and (ii) when immersed in water, as shown in Fig. 39 i. Find the apparent loss of weight. Then remove the bridge and place the beaker containing the water directly upon the balance pan, and weigh.

Suspend the same solid and the same wire from a glass rod clamped horizontally over the balance pan, and so that the solid is immersed again, and weigh the beaker. Find the apparent increase in the weight of the beaker and its contents.

Relative Density of Solids.

Relative density.—The relative density (or "specific gravity") of a substance is defined as the number of times that any volume of the substance is heavier, or lighter, than an equal volume of cold water. Hence,

$$\textbf{relative density} = \frac{\textbf{weight of substance}}{\textbf{weight of an equal vol. of water}}.$$

When the substance is a solid, and denser than water, then (by the Principle of Archimedes) this may be written

$$\textbf{relative density} = \frac{\textbf{weight of substance}}{\textbf{apparent loss of weight in water}}.$$

Expt. 43.—**Relative density of a solid denser than water.** By weighing (i) in air, and (ii) in water, find the relative density of several common solids, such as iron, brass, copper, sulphur, marble, quartz, glass, etc.

N.B.—In the case of a solid which is soluble in water, it is necessary to find its density relatively to that of some liquid in which it is insoluble and the relative density of which is known.

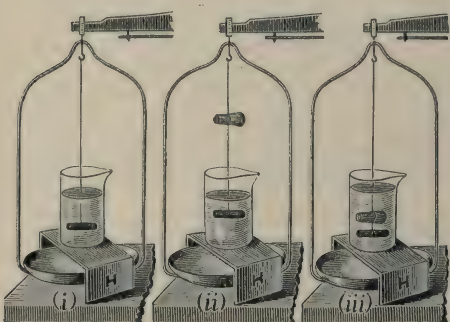


Fig. 39.—Determination of the relative density of a solid less dense than water.

Expt. 44.—**Relative density of a solid less dense than water.** Determine the relative density of a piece of cork, or of paraffin wax, in the following manner: Suspend from the balance, by means of cotton thread, a 'sinker' (a piece of lead is suitable) and find its weight when immersed in water (Fig. 39 i.).

Call this weight w_1. Support the given solid just below the hook of the balance by looping the thread once round it (Fig. 39 ii.); again weigh, and call this weight w_2. Rotate the solid downwards until it is near to the sinker (Fig. 39 iii.); again weigh, and let this weight be w_3.

The only difference between w_2 and w_3 is *the weight of the water displaced by the solid.* Hence

$$\textbf{relative density} = \frac{w_2 - w_1}{w_2 - w_3}.$$

EXPT. 44.—**Relative density of ice.** Half fill a burette, or graduated glass tube, with paraffin oil. Put the burette in a jar containing ice and water until the temperature reaches 0° C. Observe the level of the paraffin oil in the burette. Gently drop in pieces of dry ice, and observe the volume of the ice added by noticing the rise of level of the oil. Let the ice melt, and when it is converted into water, again observe the reading of the burette. This reading will give you the volume of water produced by the melting of a certain known volume of ice.

Since the weights of the ice and of the water formed are the same, their relative densities will be inversely proportional to the volumes occupied; hence

$$\textbf{relative density of ice} = \frac{\textbf{vol. of water}}{\textbf{vol. of ice.}}.$$

RELATIVE DENSITIES OF LIQUIDS.

EXPT. 45.—**Method 1. (By the principle of Archimedes.)** Weigh a glass stopper in air; then immerse it successively in water, turpentine, methylated spirit and olive oil, and notice the apparent loss of weight in each case.

The apparent loss of weight of the glass stopper in each experiment is equal to the weight of a portion of liquid of the same volume as the stopper. The numbers obtained therefore represent the weights of equal volumes of water, turpentine, methylated spirit and olive oil, and by dividing each by the number obtained in the case of water, the relative densities of the liquids are obtained.

EXPT. 46.—**Method 2. (By the 'specific gravity' bottle.)** Carefully dry the inside of the bottle, and weigh it. Fill the bottle with the given liquid, insert the stopper, *and see that no air bubbles remain in the bottle.* Dry the outside of the bottle (holding it in several folds of a duster so as not to warm the bottle by direct contact with the hand), and weigh it. Rinse out the bottle several times with water, and finally weigh it when full of cold water. In this manner you obtain the weights of *equal* volumes of the liquid and of water.

FIG. 40.—Specific gravity bottle.

Instead of the bottle, a small flask having a file mark on the neck may be used.

EXPT. 47.—**Method 3. (By the U-tube.)** (i) *If the liquids do not mix.* Turpentine or petroleum are suitable liquids to use. Fix in a vertical position a glass **U**-tube, with limbs about 40 cm. long. Half fill the **U**-tube with cold water, and pour the oil into one limb until it is nearly full. Measure the vertical distances above the table (i) of the two exposed surfaces, and (ii) of the surface of separation.

From the measurements, find the lengths of column of the water and oil which balance each other; as the less dense liquid will have the longer column, the relative densities will be *inversely* proportional to the lengths of column. Thus, if AB (Fig. 41) is the level of the surface of separation, and if h_1 and h_2 are the vertical distances above AB of the exposed surfaces of the water and of the oil respectively, and if δ_1 and δ_2 are their densities, then

$$\text{pressure at level AB} = h_1\delta_1 = h_2\delta_2,$$

or

$$\delta_2/\delta_1 = h_1/h_2.$$

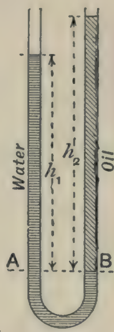


FIG. 41.

(ii) *If the liquids mix.* When the liquid of unknown relative density readily mixes with water, the two liquids must be separated by a short column of mercury. Strong brine or methylated spirit are suitable liquids. Pour mercury into a clean **U**-tube, to a depth of about 3 cm., and mark accurately with gummed paper and a pencil the level of the mercury surface. Nearly fill one of the limbs with the liquid of unknown relative density, and gradually pour cold water into the other limb until the mercury surface coincides again with the pencil mark. Measure the lengths of the two liquid columns, and calculate the relative densities.

EXPT. 48.—**Method 4. (By Hare's apparatus.)** Fig. 42 represents a convenient form of the apparatus: it consists of an inverted glass bottle with a three-holed rubber stopper. Two long glass tubes connect the interior of the bottle with the two liquids to be compared, contained in two separate beakers. The third tube terminates in a rubber tube and clip.

By applying suction to the rubber tube a partial vacuum is created in the bottle; and the liquids rise up the tubes until the pressure due to each liquid column *at the level of the liquid in its beaker* is equal to the difference between the pressure of the atmosphere and that of the partial vacuum. The height of each liquid column must be measured therefore from the surface of the same liquid in the beaker, and not from the bottom of the beaker.

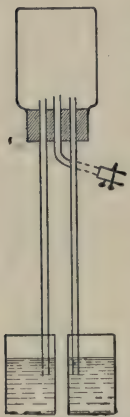


FIG. 42.—Hare's apparatus.

Take several different measurements by varying the partial vacuum in the bottle.

HYDROMETERS.

The simple apparatus used in Expt. 40 (p. 56) will serve to demonstrate the principle of a hydrometer. When placed in any liquid **the hydrometer will sink until the weight of the displaced liquid is equal to the weight of the hydrometer.** If a hydrometer, of uniform cross-section a, sinks to depths h_1 cm. and h_2 cm. in two different liquids of density δ_1 and δ_2 respectively, the weights of the liquids displaced will be $ah_1\delta_1$ gm. and $ah_2\delta_2$ gm. respectively. Hence

$$ah_1\delta_1 = ah_2\delta_2 ; \text{ or } \delta_1/\delta_2 = h_2/h_1.$$

EXPT. 49.—**Relative density, by means of a simple hydrometer.** With the apparatus of Fig. 37, find the relative density of weak brine and of dilute alcohol.

Verify the results obtained, by finding the relative densities by means of a Twaddel hydrometer (Fig. 43). The scale attached to a Twaddel hydrometer

FIG. 43.—A Twaddel hydrometer.

is arbitrary, and the relative density is calculated by the following equation:

$$\text{relative density} = 1 + \frac{\text{Twaddel degrees}}{200}.$$

Nicholson hydrometer. This is a hydrometer of *constant immersion*; and relative densities of solids and of liquids may be determined by finding the weights necessary to sink the instrument to a fixed mark on it.

EXPT. 50.—**Relative density of a solid (by Nicholson hydrometer).** Place the hydrometer (Fig. 44 i.) in a tall

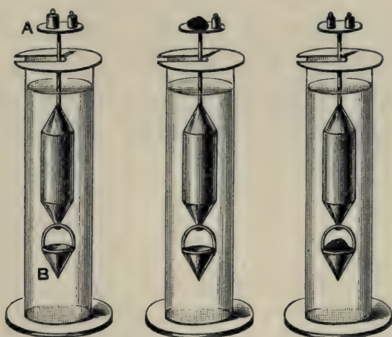


FIG. 44.—Use of a Nicholson hydrometer, to determine the relative density of a solid.

jar of water. If the instrument does not float vertically, place a small piece of lead on the lower pan (B), and keep it there during the experiment. Place weights on the top pan (A) until the instrument is immersed to the file mark on the stem; call this weight w_1. Remove the weights, place on the pan (A) the substance of unknown relative density, and add weights (w_2) until the immersion is the same as before (Fig. 44 ii.). The *weight in air* of the substance is $(w_1 - w_2)$ gm.

Remove the weights, place the substance in the pan (B), and again add weights (w_3) to the pan (A) until the immersion is the same (Fig. 44 iii.). The *weight of displaced water* is $(w_3 - w_2)$ gm. Hence, relative density of substance is $(w_1 - w_2)/(w_3 - w_2)$.

EXPT. 51.—**Relative density of a liquid (by Nicholson hydrometer).** Weigh the hydrometer and float it in a jar of water. Add weights until it is immersed to the mark on the stem. Then the *total* weight (*i.e.* the weight of the hydrometer plus the added weights) is equal to the weight of the volume of water displaced. Repeat the observation, using the other liquid. As the same volume of liquid is displaced in each case, you thus obtain the weights of equal volumes of the liquid and of water. Calculate the relative density of the liquid.

CHAPTER VII.

FORCE. FRICTION. LAW OF MOMENTS. CENTRE OF GRAVITY. SURFACE TENSION.

FORCE, MASS AND ACCELERATION.

Unit of force.—The unit of force is defined as that which, when acting freely, will give unit acceleration[1] to unit mass; and the magnitude of any force, expressed in terms of this unit, is represented by the product of the mass acted upon and the acceleration produced. Hence, the familiar equation

$$\mathbf{F} = ma,$$

where $\mathbf{F}$, m and a represent the force, the mass and the acceleration respectively.

The above fundamental equation can be demonstrated by means of an apparatus similar to Fig. 45.[2] Two aluminium pulley wheels, preferably with ball bearings, are fixed rigidly to a vertical board at least 10 ft. above the floor. A thin *silk cord*, passing over the pulleys, carries at each end a scale pan; the pans having been previously adjusted to equal weight.

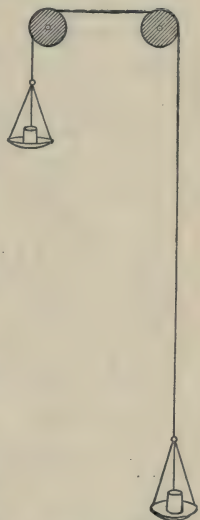


FIG. 45.—Apparatus for Expt. 52.

[1] Acceleration is defined as the change of velocity in unit time.

[2] More accurate results can be obtained by means of either Fletcher's Trolley Apparatus or Cusson's Patent Ribbon Attwood's Machine. Both are made by Messrs. G. Cussons, Ltd., Broughton, Manchester; and full instructions for using them are supplied by the maker.

The length of the cord is adjusted so that either pan can fall through a vertical distance of about 8 ft. With this apparatus it can be proved that (i) **when the mass is constant, the acceleration is proportional to the force**, and (ii) **when the force is constant, the acceleration is inversely proportional to the mass moved.** Taking s to represent the distance (in ft.) through which the body falls, a the acceleration (in ft. per sec. per sec.), and t the time in seconds, the acceleration is calculated from the formula $s = at^2/2$, after measuring the vertical distance through which the heavier pan falls and the time occupied.

EXPT. 52.—Having fitted up the apparatus as in Fig. 45, place equal weights, *e.g.* 2 lb., in each pan. Raise the left pan to its highest position, and measure the vertical distance from the pan to the floor.[1] Add a small excess weight, *e.g.* 0·5 oz., to the left pan, and test whether this is sufficient to overcome the friction in the pulleys: this is done by giving to the right-hand pan a slight upward velocity, and observing whether the motion continues with *constant velocity.* Alter the excess weight until this constant velocity is obtained. (This must be done with considerable care; and, if British units are used, it will be found convenient to have a number of weights each equal to 0·1 oz.) This excess weight must be kept in the pan during the whole experiment.

Now add another weight, *e.g.* 0·5 oz., to the left pan: this will be the *force* causing the motion. Raise the pan to its highest position; with one hand hold the cord above the right pan, and in the other hold a stop watch. Release the cord and start the watch at the

[1] It is advantageous to place on the floor and below the left pan a shallow tray (*e.g.* the lid of a cardboard box) containing sand, the surface of which has been pressed to the same shape as the pan. This will prevent the weights from being jerked out of the pan at the end of the motion.

same instant, and find the time required by the left pan to reach the floor. Repeat the observation at least four times. From the average of these, and from the vertical distance, calculate the acceleration.

In the same manner determine the acceleration when the force causing motion is increased to 1 oz. From the results obtained, determine whether the acceleration is proportional to the force. (It is assumed that the mass has remained constant, whereas in the second observation the mass was greater by 0·5 oz.; the error, however, amounts to much less than 1 %, and it may be disregarded.)

Repeat the observations when heavier weights, *e.g.* 3 lb., are placed in each pan. It will be necessary to re-determine the small excess weight necessary to overcome friction. Find the acceleration when the force causing motion is 1 oz. weight. Compare the result with the preceding observation when the force was the same, and find whether the acceleration is inversely proportional to the total mass.

The following data indicate the degree of accuracy obtainable with a simple arrangement of apparatus :

EXAMPLE 1.—**Force and acceleration.**
Vertical distance (s) moved, 106 in. (8·83 ft.).
Total weight on right, 2·253 lb.
" " left, 2·253 lb.
Weight required to overcome friction, 0·053 lb.
(i) Moving force ($\mathbf{F}_1$) = 0·5 oz. = ·0312 lb.

$$\text{Time } (t_1) \text{ of fall, } \left.\begin{array}{l} 10\cdot3 \\ 10\cdot2 \\ 10\cdot3 \\ 10\cdot4 \end{array}\right\} \text{average, } 10\cdot3 \text{ sec.}$$

$$\text{Acceleration } (a_1) = \frac{2s}{t_1^2} = \frac{17\cdot66}{(10\cdot3)^2} = 0\cdot166 \text{ ft./sec}^2.$$

(ii) Moving force ($\mathbf{F}_2$) = 1 oz. = 0·0625 lb.

$$\text{Time } (t_2) \text{ of fall, } \left.\begin{matrix} 7{\cdot}2 \\ 7{\cdot}2 \\ 7{\cdot}4 \\ 7{\cdot}2 \end{matrix}\right\}\text{average, } 7{\cdot}25 \text{ sec.}$$

$$\text{Acceleration } (a_2) = \frac{2s}{t_2^2} = \frac{17{\cdot}66}{(7{\cdot}25)^2} = 0{\cdot}336 \text{ ft./sec}^2.$$

Comparing Experiments (i) and (ii),

$$\frac{\mathbf{F}_2}{\mathbf{F}_1} = \frac{1 \text{ oz.}}{0{\cdot}5 \text{ oz.}} = 2,$$

$$\frac{a_2}{a_1} = \frac{0{\cdot}336}{0{\cdot}166} = 2{\cdot}02.$$

EXAMPLE 2.—**Mass and acceleration.**

Vertical distance (s) moved, 105 in. (8·75 ft.).
Total weight on right, 3·253 lb.
" " left, 3·253 lb.
Weight required to overcome friction, 0·075 lb.
Moving force ($\mathbf{F}$) = 1 oz. = ·0625 lb.

$$\text{Time } (t_3) \text{ of fall, } \left.\begin{matrix} 8{\cdot}6 \\ 8{\cdot}7 \\ 8{\cdot}5 \\ 8{\cdot}7 \\ 8{\cdot}7 \end{matrix}\right\}\text{average, } 8{\cdot}64 \text{ sec.}$$

$$\text{Acceleration } (a_3) = \frac{2s}{t_3^2} = \frac{17{\cdot}50}{(8{\cdot}64)^2} = 0{\cdot}234 \text{ ft./sec}^2.$$

Compare this with Example 1 (ii). If M_2 and M_3 are the *total* masses moved in the two observations respectively, then

$$M_2 = 4{\cdot}559 + 0{\cdot}062 = 4{\cdot}621 \text{ lb.},$$

and $$M_3 = 6{\cdot}581 + 0{\cdot}062 = 6{\cdot}643 \text{ lb.}$$

Also $$a_2 = 0{\cdot}336 \text{ ft./sec}^2\text{., and}$$
$$a_3 = 0{\cdot}234 \quad " \quad .$$

Whence $$M_3/M_2 = 6{\cdot}643/4{\cdot}621 = \mathbf{1{\cdot}43}\,(7);$$

and $$a_2/a_3 = 0{\cdot}336/0{\cdot}234 = \mathbf{1{\cdot}43}\,(6).$$

Rough determination of "g."—The apparatus and method of Expt. 52 may be used for determining the **acceleration due to gravity** (or "g"). But, chiefly owing to the difficulty of neutralising accurately the effect of friction, the result is only approximate, and not nearly so trustworthy as that obtained by means of the simple pendulum (see below).

The acceleration g may be defined as that imparted to a mass when it is acted upon by a force equal to the weight of the mass. Using the symbols of the preceding experiments, and since **the acceleration is proportional to the force**, the ratio of g to a will be equal to the ratio of M to **F**; or

$$\frac{g}{a}=\frac{\mathrm{M}}{\mathbf{F}}, \quad \text{or} \quad g=\frac{\mathrm{M}a}{\mathbf{F}}.$$

The following data indicate the degree of accuracy which may be expected:

Total mass (M) = 6·68 lb.
Force (**F**) = 0·0937 lb.
Distance (s) = 8·73 ft.
Average time (t) of fall = 6·8 sec.
Acceleration = 0·378 ft./sec^2.
Hence, $g = (6{\cdot}68 \times 0{\cdot}378)/0{\cdot}0937 = 27{\cdot}1$ ft./sec^2.

Determination of "g" by the simple pendulum.—The relation between **t** (the time of vibration of a simple pendulum), **l** (the length of the pendulum), and **g** (the acceleration due to gravity) is expressed by the equation

$$\mathbf{t}=\pi\sqrt{\frac{\mathbf{l}}{\mathbf{g}}}, \quad \text{or} \quad \mathbf{g}=\frac{\pi^2\mathbf{l}}{\mathbf{t}^2}.$$

The value of g therefore can be calculated when **l** and **t** are known. In the observations of Expt. 9 (p. 14) it was found that $t = 1{\cdot}14$ sec. when $l = 130$ cm.; hence

$$g=\frac{\pi^2 \times 130}{(1{\cdot}14)^2}=987 \text{ cm./sec}^2.$$

This result is about 0·6 % too high.

The value of g varies from 978 cm./sec². at the Equator to 983 cm./sec². near the Poles. In the British Isles the value is about 981 cm./sec²., in India about 978 cm./sec²., and in Africa and Australasia about 979 cm./sec²., but the exact value varies according to locality.

The parallelogram of forces.—The principle of the parallelogram of forces may be expressed thus: **If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant of these two forces will be represented in magnitude and direction by that diagonal of the parallelogram which passes through the same point.**

EXPT. 53.—**Demonstration of the principle of the parallelogram of forces.** Lay a large sheet of paper on

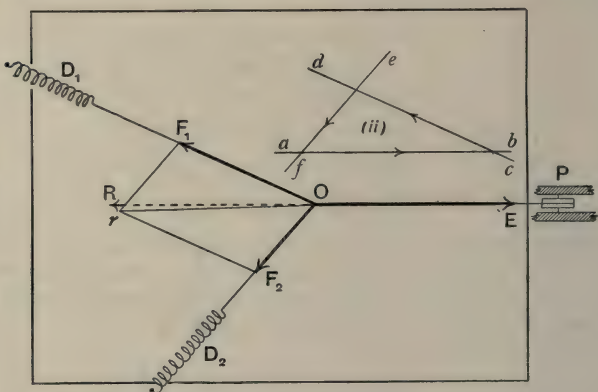


FIG. 46.—Experiment on the parallelogram of forces.

a flat table (Fig. 46). Connect three threads together at a point O. Pass one of the threads over a pulley wheel P, clamped firmly near to the edge of the table, and fasten a known weight to the end of the thread. Join the other threads to spring dynamometers, D_1 and

D_2, to which graduated scales are attached. Allow the known weight to hang freely.

The point O is now in equilibrium under the action of the *two forces* (exerted by the dynamometers) and their *equilibrant* (which, in this case, is the weight hanging over the pulley). Mark on the paper, by means of a needle point, the direction of the three threads; and write down the magnitude of the three forces.

Remove the paper, and mark off the lengths OE, OF_1 and OF_2 proportional to the three forces. Complete the parallelogram, constructed with OF_1 and OF_2 as adjacent sides; and draw the diagonal O*r*. According to the principle of the parallelogram of forces, this diagonal should be equal in magnitude and opposite in direction to the equilibrant OE. Produce OE backwards, and mark off a length OR equal to OE. If the experiment is carried out with care the lines OR and O*r* should be identical in length and in direction.

(ii) Alter the directions of the forces OF_1 and OF_2, and again observe whether the true resultant coincides with that deduced by geometrical construction.

EXPT. 54.—**The triangle of forces.** On the same sheet of paper used in Expt. 53 draw three lines *ab*, *cd*, and *ef* parallel to the forces OE, OF_1 and OF_2 respectively (Fig. 46). These lines enclose a triangular area. Measure the lengths of the sides of the triangle thus obtained. These three lengths will be found to be proportional to the three forces respectively. Mark, by means of arrow-heads, the directions of the forces.

It is evident, therefore, that **if three forces acting at a point are in equilibrium they can be represented in magnitude and direction by the sides of a triangle taken in order.** This is known as the principle of **the Triangle of Forces.**

EXPT. 55.—**The polygon of forces.** Place a sheet of paper on a table or bench. Tie four threads together at a point O (Fig. 47), and attach the free ends to four separate spring dynamometers. Fasten the upper ends of the latter to different points of the table, adjusting the tension of each so that a suitable elongation of the springs is obtained. Let the forces exerted by the springs be denoted by F_1, F_2, F_3, F_4. Mark on the paper the directions of these forces; and mark off lengths OF_1, OF_2, etc., proportional to their magnitudes. The point O is now in equilibrium under the action of the four forces represented.

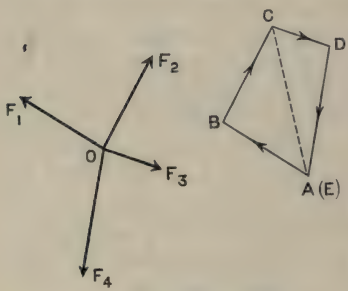


FIG. 47.—The polygon of forces.

As in Expt. 54, construct a *force-diagram* by drawing lines AB, BC, CD and DE parallel to, and proportional to the magnitudes of, the forces F_1, F_2, etc. Mark with arrow-heads the direction of the forces. Notice whether the point E coincides with the point A. If the measurements are accurate, these points will coincide, and the force-diagram will completely enclose a space.

This experiment is an application of the principle of **the polygon of forces**, which may be stated thus:—**If any number of forces acting at a point are in equilibrium, they may be represented in magnitude and direction by the sides of a polygon taken in order.** It is seen readily that this is simply an extension of the principle of the triangle of forces. For, in Fig. 47, if the points A and C are joined, CA represents the equilibrant, and AC represents the resultant of AB and BC. Thus the force-diagram is reduced to the triangle ACD; and DA is the equilibrant, and AD is the resultant, of the forces AC and CD.

Coefficient of friction.—When a rectangular block is resting on a horizontal surface, a small force may be applied horizontally to the block without causing it to move. When this force is increased *gradually* to a certain maximum value, the block will begin to move. This maximum force measures what is termed the **limiting friction**.

The **coefficient of friction** (μ) between the given surfaces is defined as the ratio

$$\frac{\text{limiting friction } (\mathbf{P})}{\text{pressure between the surfaces } (\mathbf{W})}.$$

EXPT. 56.—**Coefficient of friction between wood and wood.** For the purpose of this experiment it is convenient to

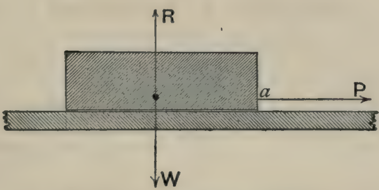


FIG. 48.—Experiment on friction.

have a block of wood, about 15 cm. × 10 cm. × 5 cm., and a stout board of the same material, about 70 cm. × 12 cm. × 3 cm. Both of these should be accurately planed. Fix a small staple into a middle point *a* (Fig. 48) of one of the smaller faces of the first block, and weigh it. By means of a spirit level adjust the long board so that its surface is horizontal. Attach a spring dynamometer to the staple and gradually apply an increasing horizontal force (**P**). Note the reading of the dynamometer at the instant when motion is just beginning. Repeat the observation several times, and note the average value.

Increase the load **W** by placing weights of 100, 200, ... grams on the top of the block; and measure for each load the limiting friction. Tabulate the observations thus:

Material of the surfaces, e.g. mahogany on mahogany.

Total Load (**W**).	Limiting Friction (**P**).	Coefficient of friction $(\mu)=P/W$.
(i)	....	....
(ii)	....	....
(iii)	....	....

EXPT. 57.—**Friction is independent of area of contact surfaces.** Using the same apparatus as in Expt. 56, turn the block so that it now rests on one of its smaller faces. Determine whether, with the same *total loads* as before, the limiting friction is also the same.

The Law of Moments.—The moment of a force about any point is defined as the product obtained by multiplying the force by the perpendicular distance between the point and the line of action of the force.

It is agreed generally that the moment of a force which tends to produce motion in an anti-clockwise direction shall be regarded as positive; and negative if the direction is clockwise.

The Law of Moments may be stated thus: **When a body, free to rotate about a given point, is in equilibrium under the action of two or more forces, the algebraic sum of the moments about this point is equal to zero.**

EXPT. 58.—**Law of Moments.** In Fig. 49 AB is a uniform rod of wood, 36 in. $\times \frac{3}{4}$ in. $\times \frac{1}{2}$ in., with a number of round holes 2 inches apart bored through the rod

along a middle line. Suspend the rod in front of a vertical board by means of a round nail at A, so that the rod can swing freely. Attach three pulley-wheels (P_1, P_2, P_3) to the board. Fasten the ends of three pieces of thin flexible string, or silk cord, to different points of the rod; the small attached diagram (Fig. 49) suggests a convenient method of

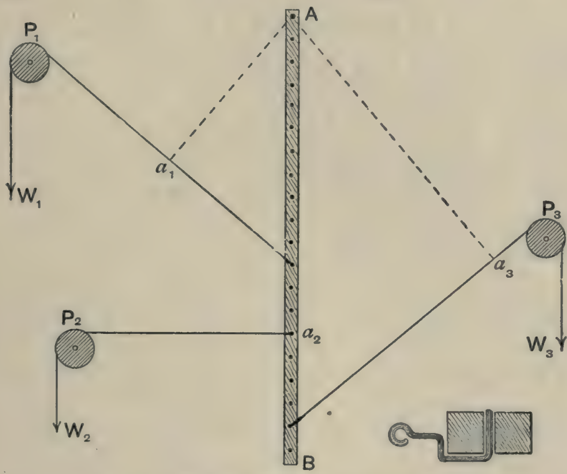


FIG. 49.—Experiment on the law of moments.

attaching, by means of bent thick wire, the strings to various points on the rod. Attach weights to the free ends of the strings, and adjust the weights so that the rod remains in equilibrium in a vertical position.

Mark, with the help of a **T**-square, on the board or on a sheet of paper pinned to the board the perpendicular distances Aa_1, etc. These distances are the *arms* of the forces round the point A.

Record your observations thus:

Positive Moments.	Negative Moments.	Algebraic Sum of Moments.
$(W_3 \times Aa_3) = + \ldots\ldots$	$W_1 \times Aa_1 = - \ldots\ldots$ $W_2 \times Aa_2 = - \ldots\ldots$	....

Repeat the observation at least three times, using different points of attachment for the forces.

Reactions at the ends of a loaded beam.—Let AB (Fig. 50) represent a uniform beam supported at each end, and loaded irregularly by weights W_1 and W_2 suspended from points a_1 and a_2. For simplicity, assume that the weight of the beam is negligibly small. Since the beam remains at rest, the sum of the two upward forces, or *reactions* R_1 and R_2, exerted by the supports, must be equal to the sum of the downward forces; hence

$$R_1 + R_2 = W_1 + W_2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Also, the beam may be imagined to be pivoted *at any point* and capable of rotation round that point. Since the beam is in equilibrium, the algebraic sum of the moments of all the forces round that point must be equal to zero. Suppose that the beam is pivoted at the point A; then

$$(R_2 \times AB) - \{(W_1 \times Aa_1) + (W_2 \times Aa_2)\} = 0. \quad \ldots\ldots\text{(ii)}$$

All quantities in this equation, except R_2, are known; hence the magnitude of R_2 can be calculated. Finally, R_1 can be calculated from equation (i).

EXPT. 59.—**Determination of reactions.** Support a uniform wooden rod by means of two compression balances (Fig. 50). A suitable method is to remove the scale pans and place a triangular file across the top of each balance. If the pointers do not reach the zero of the scales, rectify this by adding pieces of lead, or other weights, to the top of the scales. Suspend suitable

weights W_1 and W_2 from any two points a_1 and a_2. Note the weights, and measure the distances AB, Aa_1 and Aa_2.

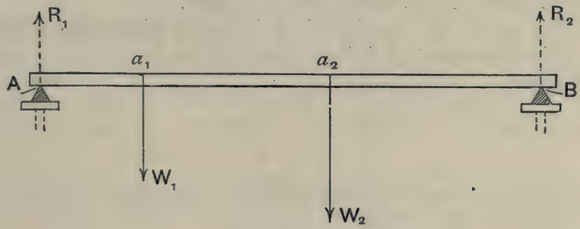


FIG. 50.—The reactions at the ends of a loaded beam.

Calculate the values of R_1 and R_2 thus:

(i) $W_1 + W_2 = \ldots\ldots$
(ii) $R_2 = \{(W_1 \times Aa_1) + (W_2 \times Aa_2)\}/AB = \ldots\ldots$
(iii) $R_1 = (W_1 + W_2) - R_2 = \ldots\ldots$

Verify these results by observing the readings of the balances, thus:

(iv) (Balance A) $R_1 = \ldots\ldots$
(v) (Balance B) $R_2 = \ldots\ldots$

Repeat the experiment, using different weights and different points of suspension.

The resultant of parallel forces.—In Fig. 51, AB represents a *uniform* wooden rod from which two known weights W_1 and W_2 are suspended. When the rod is uniform, its own weight w may be regarded as a third force acting at its middle point g. There are then three parallel forces (W_1, W_2 and w) acting downwards. A point O can be found at which the rod can be supported in equilibrium. The single upward force E, which can be called the equilibrant, must be equal and opposite to the **resultant** of the three downward forces. Also we may imagine that the rod is free to rotate round the point O; and since it remains in equilibrium, the point O must be such that the algebraic

sum of the moments of all the forces round O is equal to zero. Hence we derive the following laws defining the resultant of unequal parallel forces:

(i) The magnitude of the resultant is equal to the algebraic sum of the component forces.

(ii) Its point of application is such that the algebraic sum of the moments of the components round that point is equal to zero.

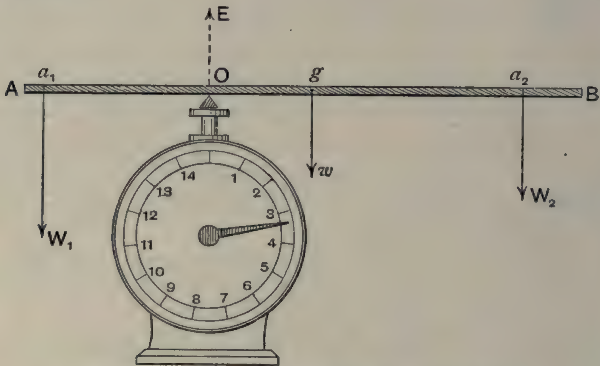


FIG. 51.—The resultant of two or more like parallel forces.

EXPT. 60.—**Measurement of resultant of unequal parallel forces.** Find the weight w of a uniform wooden rod AB (Fig. 51), and mark its middle point g. Suspend two known weights W_1 and W_2 from any two points a_1 and a_2. Support the rod on a knife-edge (*e.g.* the edge of a triangular file) resting on the top of a compression balance. Adjust the position of the point of support so that the rod remains in equilibrium. Measure the distances ga_1 and ga_2, and calculate the distance Og.

Thus, taking moments round O.

$$W_1 \times (ga_1 - Og) - \{(w \times Og) + W_2(Og + ga_2)\} = 0\,;$$

from this equation find the length Og. Finally read the balance and measure Og. Record your results thus:

(By theory) (i) Resultant $=(W_1+W_2+w)=$

(ii) $Og=$

(By experiment) (i) Resultant $=E=$

(ii) $Og=$

Resultant of two unlike parallel forces.—Fig. 52 represents a wooden rod suspended in front of a vertical board by means of a loop of string and a nail. Two parallel forces W_1 and W_2, of which W_1 is greater than W_2, are applied by means of strings and pulleys to any two points a_1 and a_2. In order that the rod may be held in its vertical position of equilibrium it is necessary to apply a third force E, acting in the same direction as the smaller force W_2, and of such magnitude that $(E+W_2)=W_1$. Also, since we may imagine that the rod is pivoted at O, and capable of rotating round that point, the sum of the moments round O of W_1 and W_2 must be zero. The resultant is a force equal and opposite to E, and acting at the point O. Hence, in the case of two unlike parallel forces,

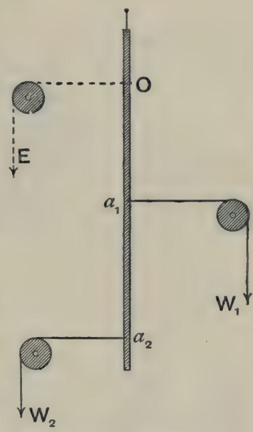


FIG. 52.—The resultant of unlike parallel forces.

(i) the resultant is equal to the difference between the component forces, and in a direction parallel to them; and

(ii) the resultant acts at a point such that the algebraic sum of the moments of the components round that point is equal to zero.

EXPT. 61.—**Measurement of resultant of unlike parallel forces.** Set up the apparatus shown in Fig. 52, and apply the two parallel forces W_1 and W_2. By means

of a third length of string and a pulley wheel apply another force E, equal in magnitude to $(W_1 - W_2)$; this pulley wheel should be mounted so as readily to modify its position. Adjust the pulley until the point of application of E is such that, when the string is horizontal, the rod is restored to its vertical position.

Measure the distances Oa_1 and Oa_2, and calculate the algebraic sum of the moments

$$(W_1 \times Oa_1) - (W_2 \times Oa_2).$$

Equilibrant of a couple. When the two parallel and unlike forces are equal in magnitude (*e.g.* when W_1 and W_2, in Fig. 52, are equal) the equilibrant is equal to zero. There is no single force, therefore, which will restore the rod to equilibrium.

Nor is there any point in the rod round which the algebraic sum of the moments is equal to zero. If each force is W, and O is any point in the rod, the algebraic sum of the moments of the forces is

$$(W \times Oa_1) - (W \times Oa_2),$$

or

$$W(Oa_1 - Oa_2),$$

or

$$W \times a_1 a_2.$$

The sum of the moments, therefore, is independent of the position of the point O. Such a pair of forces is termed a **couple**; and the moment of a couple is equal to the product of one of the forces and the perpendicular distance between their lines of action.

A body acted upon by a couple can be kept in equilibrium only by applying to it another couple of the same moment and acting in the opposite direction.

EXPT. 62.—**Determination of the equilibrant of a couple.** Suspend a rod (Fig. 53) and apply two equal parallel forces (W and W) to any two points a_1 and a_2. By means of two additional strings and pulley-wheels apply two other equal forces (E and E) at such points b_1 and b_2

that the latter couple tends to produce rotation in the opposite direction to that due to the couple WW. Carefully adjust the position of the point b_1 until, when the force applied at b_1 is horizontal, the rod returns to its position of equilibrium.

Measure the distances a_1a_2 and b_1b_2, and calculate the moments of the two couples, viz. $(W \times a_1a_2)$ and $(E \times b_1b_2)$.

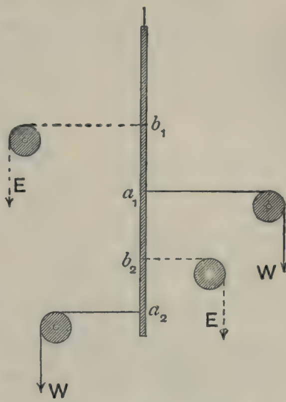


FIG. 53.—The equilibrant of a couple.

Centre of gravity.—In any rigid body the weights of all the small particles or fragments into which the body may be divided can be regarded as a *system of parallel forces*, all acting vertically downwards. These parallel forces can be replaced by a single resultant force, also acting vertically downwards and equal in magnitude to the sum of the component forces. **The point of application of this resultant force is termed the centre of gravity of the body.** The same point may be defined as the point of application of the **equilibrant** of the system of parallel forces. Hence, when the rigid body is supported at its centre of gravity it will remain in equilibrium: this serves as an experimental method of verifying the position of the point.

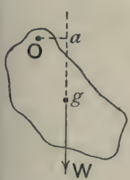


FIG. 54.

The experimental method of finding the centre of gravity is based upon the fact that when the rigid body is supported from a single point (O, Fig. 54) round which it is free to rotate, the resultant force W acting at the centre of gravity g will exert a turning moment $(W \times Oa)$ which will cause rotation until the point g is vertically below O.

EXPT. 63 (i).—**Centre of gravity of a triangular lamina.** Cut out from a sheet of thick cardboard a large triangle. Pierce a pinhole near one angle of the triangle, and suspend the triangle in front of a vertical drawing board by means of a pin passing through the hole. Suspend from the same pin a length of cotton carrying a piece of lead at its lower end. See that the cardboard and plumb-line are quite free to rotate; then, when both are at rest, mark with a pencil point the direction of the cotton along the cardboard. Repeat this, using a corresponding hole near another of the angles. Dismount the apparatus, and draw with a straight edge the two lines obtained. As the centre of gravity is somewhere along both lines it must be where these lines intersect.

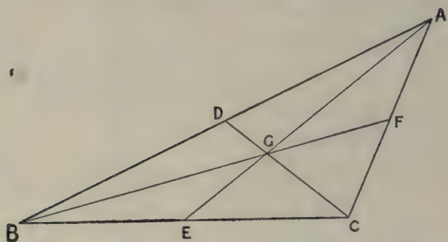


FIG. 55.—The centre of gravity of a triangular lamina.

Verify the result by attempting to balance the cardboard on the sharp point of a pencil held vertically.

Also verify the result geometrically, by joining the middle point of each side to the opposite angle (Fig. 55): the point of intersection of these median lines is the centre of gravity.

EXPT. 63 (ii).—Lay a uniform lamina, similar to that used in Expt. 63 (i), flat on a square-edged table or bench. Gradually slide it forward so as to project beyond the edge, until it is just about to topple over. Hold it firmly in position and draw a pencil-line on the undersurface and coinciding with the edge of the

table. Rotate the lamina through a right angle and repeat the process. The point of intersection of the two pencil-lines coincides with the centre of gravity.

Resultant centre of gravity.—In Fig. 56 two squares of cardboard of the same thickness are glued together. By geometry, the centres of gravity of the squares will be at their middle points, g_1 and g_2. Since the weights of the squares are proportional to their areas, they may be represented numerically by a^2 and b^2, where a and b are the lengths of one edge of each square respectively. We may imagine therefore that two weights, a^2 and b^2, are acting respectively at two points g_1 and g_2. The resultant of these two weights will act at a point G, on the line g_1g_2, such that

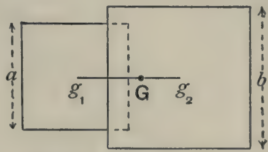


FIG. 56.—Resultant centre of gravity.

$$a^2 \times Gg_1 = b^2 \times Gg_2.$$

Hence $$a^2 \times Gg_1 = b^2(g_1g_2 - Gg_1),$$

or $$Gg_1 = g_1g_2 \times \frac{b^2}{a^2+b^2}.$$

EXPT. 64.—**Centre of gravity by experiment and geometry.** Construct a cardboard model of Fig. 56, and find its centre of gravity experimentally by suspending it consecutively from two different points, as in Expt. 63. Verify the result by calculating the area of each square and the position of the point G, as explained above.

EXPT. 65.—**Centre of gravity of a hollow enclosed cone.** Mark on the slant surface of the cone any straight line OC (Fig. 57 i), and paste along this line a paper scale divided into inches and tenths; for this purpose, a strip may be cut from squared paper suitably divided. Mark another point B on the circumference of the base, and at a distance from C equal to one-fourth of the circum-

ference. At any two points, such as A and B, on the line OB fix small screw-hooks. Attach strong thread to the hook at A, and suspend the cone from a horizontal rod. Suspend a plumb-line from the same rod, and just in front of the cone. View the arrangement

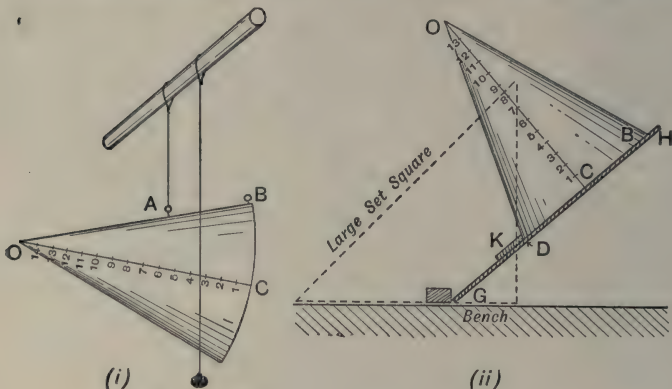


FIG. 57.—The centre of gravity of a hollow cone.

horizontally, and with one eye only; move the eye until the plumb-line coincides with the thread attached to A, and adjust the cone so that its axis is in a plane perpendicular to the line of sight. Note the reading of the plumb-line on the scale OC. Suspend the cone from the hook at B, and note whether the same scale-reading is obtained.

Since the centre of gravity must be (i) on the vertical plane passing through the scale-reading obtained as above, and (ii) also on the axis of symmetry of the cone, it must be situated at the point where these intersect.

Fig. 57 ii suggests how this result may be verified by an independent method. Arrange a smooth board GH so that it can be tilted round the edge G, and fasten to it a

narrow wood strip K parallel to the edge G. Place the cone on the board so that a point D, diametrically opposite to B, touches the strip K. Slowly tilt the board, and hold in front of the apparatus a large set-square, always adjusting its position so that its vertical edge coincides with the upper edge of the strip K. As soon as the centre of gravity of the cone moves to the left of the vertical line through D, the equilibrium becomes unstable, and the cone tumbles over. Very slowly increase the slope of the board and, observing the scale with one eye only and with the line of sight perpendicular to the set-square, note the scale-reading of the edge of the set-square just at the instant when the cone tumbles over. This scale-reading should coincide with that obtained by the previous experiment.

When the cone is hollow and open (*i.e.* not provided with a base), from theory the centre of gravity would be situated on the axis of symmetry and at a distance from the base equal to one-third of the height of the cone. When the cone is provided with a base, the centre of gravity will be situated, of course, at a point nearer to the base. Assuming that the material is uniformly thick, the position of the centre of gravity can be calculated by determining the area (i) of the slant surface and (ii) of the base.

EXPT. 66.—**The centre of gravity of a wickerwork-basket.** Suspend the basket by thread attached to any point on the edge or side of the basket, and hang a plumb-line from the same point, adjusting the plumb-line so that it hangs freely and vertically through some opening in the wickerwork. Mark the position of the plumb-line, and tie a piece of thread across the basket and in the position occupied by the plumb-line. Suspend the basket from some other point, and repeat the procedure. The point of intersection of the two threads is the centre of gravity.

Surface Tension.

Surface tension of water.—When a narrow capillary tube is held vertically with its lower end immersed in clean water, the water rises up the tube to a height depending upon the diameter of the tube—the narrower the tube the greater the height. The weight of the liquid column above the level of the surrounding water is supported by the **surface tension** of the water round the edge of the meniscus. If the tension is T dynes per cm. length, and if r is the radius of the tube, then, since the tension is vertical, the total upward force is $2\pi r\mathrm{T}$. If h is the height of the liquid column, and ρ gm. per c.c. the density of the liquid, the weight of the liquid column is $\pi r^2 h\rho$ gm.

Hence $$2\pi r\mathrm{T} = \pi r^2 h\rho \quad (\text{or } \pi r^2 h\rho g \text{ dynes}),$$

or $$\mathrm{T} = \frac{hr\rho g}{2} \text{ dynes per cm.}$$

The surface tension of water depends upon the temperature; the higher the temperature the less the tension. This is expressed by the equation

$$\mathrm{T}_t = \mathrm{T}_0 - \beta t,$$

where T_t and T_0 are the tensions at $t°$ C. and 0° C. respectively, and $\beta = 0{\cdot}152$. The value of T_0, for water, is 75·8 dynes per cm.

Expt. 67.—**Measurement of surface tension.** Select two, or more, pieces of thermometer tubing with circular bore, the widest not more than 0·5 mm. diameter. Test the uniformity of bore of each piece by drawing into it a short thread (10-12 cm.) of *clean* mercury; measure the length of the mercury thread at several different parts of the tube; if the length varies appreciably, discard the tube. If the length varies but slightly, calculate the average length, transfer the whole of the thread to a previously weighed watch glass, and again weigh. From

the known density of mercury, calculate the average radius of the tube.

Clean the tubes by drawing into them a thread of concentrated nitric acid, and wash them by running through the tubes a stream of *tap* water. *Do not use distilled water.* Clean a deep beaker, and finally cleanse it by running tap water into it for several minutes.

The method shown in Fig. 58 for measuring the height of the water column consists of an accurately divided steel scale with a needle fixed to one end with lumps of soft wax. Measure accurately with calipers the distance from the end of the scale to the point of the needle. Clamp one of the tubes, and the scale, in a vertical position, as shown, and wet the higher parts of the tube by momentarily raising the beaker. Finally adjust the scale so that the needle-point just touches the surface of the water.

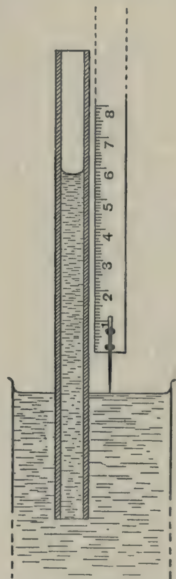


Fig. 58.—Measurement of surface tension.

Read the height of the water column, and also the temperature of the water. Calculate the value of T from the equation $T = hr\rho g/2$; and verify your result by the equation $T_t = 75{\cdot}8 - (0{\cdot}152 \times t)$. Repeat the experiment, using a tube of different internal diameter. It is evident from the above equation that, when the temperature is the same, the height of the water column varies inversely as the radius of the tube.

The procedure outlined above should give results differing from the theoretical value by not more than 1 %.

CHAPTER VIII.

MACHINES.

Machines.—A machine is any contrivance by which a force acting at a given point and in a given direction may be rendered available at some other point and in some other direction. An inclined plane, pulley blocks, the wheel and axle, a waterwheel, a windmill, etc., are examples of machines.

It is convenient in considering any machine to refer to the applied force as the *force* (F), and to the resistance overcome as the *load* (W). The work done by any force is expressed by the product of the force and the distance through which its point of application moves in the direction of the force; hence, the work put into a machine is equal to Fs_1 units, where s_1 is the distance through which the point of application of F moves. Similarly, the work derived from the machine is equal to Ws_2 units, where s_2 is the distance through which the load is moved. In actual practice a part of the work done by the force is used up in overcoming the friction of the working parts of the machine, and Fs_1 is always greater, therefore, than Ws_2. The ratio of the latter to the former is termed the **efficiency of the machine**; or

$$\textbf{efficiency} = \frac{\text{work done on load}}{\text{work done by force}} = \frac{Ws_2}{Fs_1}.$$

The **mechanical advantage** or **force-ratio** is the term applied to the ratio of the load to the force; or

$$\textbf{mechanical advantage} = \frac{\text{load}}{\text{force}} = \frac{W}{F}.$$

The **velocity ratio** of a machine is the ratio of the

distance moved through by the force to that moved through by the load; or

$$\textbf{velocity ratio} = \frac{\text{distance moved through by the force}}{\text{distance moved through by the load}} = \frac{s_1}{s_2}.$$

The inclined plane.—A suitable form of apparatus consists of a board, about $2\frac{1}{2}$ ft. long, hinged at one end to a base-board and with its upper surface as smooth as possible. It is desirable that the upper surface consist of a slab of plate glass, as shown in Fig. 59. The load W may be

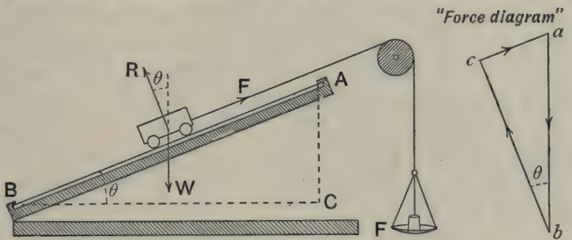


FIG. 59.—The inclined plane.

either a solid metal roller or a wooden truck fitted with wheels. The force F is transmitted by means of a string passing over an adjustable pulley wheel.

EXPT. 68.—**Principle of the inclined plane.** Weigh the load W and the pan to be used for applying the force F. Set the inclined plane to an angle of about 30°. Adjust the pulley wheel so that the string is parallel to the inclined plane, and adjust the weights in the pan so that the load is exactly balanced. As there is unavoidable friction, the weights in the pan should be such that on giving a slight push to the truck it travels with equal readiness up the plane and down the plane; or, the following procedure may be followed: gradually increase F until the truck begins to move *up* the plane, then gradually diminish F until the truck begins to move *down* the plane, and take the mean of these two values of F.

Measure the length AB of the inclined plane, and find the height of the plane by measuring the vertical heights of A and of B above the bench surface.

The load is in equilibrium under the action of *three* forces, viz. the force F, the load W and the reaction R of the surface of the inclined plane. **These forces, therefore, may be represented in magnitude and direction by the three sides of a triangle taken in order.** From the measurements of height and length draw a triangle to represent accurately the slope of the inclined plane, and insert the directions of the forces W, R and F. With the aid of this figure construct a **force diagram,** as shown in Fig. 59, by drawing a vertical line *ab*, its length indicating to a suitable scale the weight of the load W. Then draw *bc* parallel to the reaction R, and *ac* parallel to F. To the same scale, *ca* represents the force and *bc* represents the reaction of the plane. It is evident that the triangles *abc* and ABC are similar; hence

$$\frac{\mathrm{F}}{\mathrm{W}}=\frac{ac}{ab}=\frac{\mathrm{AC}}{\mathrm{AB}}=\frac{\text{height of plane}}{\text{length of plane}}.$$

Record your observations thus:

Expt. i.

$$\left.\begin{aligned}\text{Force}&=\ldots\ldots\\ \text{Load}&=\ldots\ldots\end{aligned}\right\}\text{Ratio, } \frac{\mathrm{F}}{\mathrm{W}}=\ldots\ldots$$

$$\left.\begin{aligned}\text{Height}&=\ldots\ldots\\ \text{Length}&=\ldots\ldots\end{aligned}\right\}\text{Ratio, } \frac{h}{l}=\ldots\ldots$$

Efficiency and mechanical advantage.—Increase the force (F) until it is sufficient to pull the load up the inclined plane *with constant velocity*. Assuming that the load is pulled up the entire length of the plane, the work done *on* the load is equal to $(\mathrm{W}\times h)$ units, and the work done by the force is equal to $(\mathrm{F}\times l)$ units where h is the vertical distance moved and l the length moved. Hence,

$$\textbf{efficiency}=\frac{\mathrm{W}\times h}{\mathrm{F}\times l}=\ldots\ldots$$

Similarly, the **mechanical advantage** $=\frac{\mathrm{W}}{\mathrm{F}}=\ldots\ldots$

Pulleys.—Fig. 60 represents a system of pulley wheels in which three loose pulleys and one fixed pulley are connected by three separate strings. Let W be the load, F the force applied; T_1, T_2 and T_3 the tensions in the three strings; w_1, w_2 and w_3 the weights respectively of the three loose pulleys. Consider the tensions in the strings, beginning with the lowest. The total load supported by this string is $(W+w_1)$, and this load is divided equally between the two portions of the string; hence $T_1=\frac{1}{2}(W+w_1)$. The total load supported by the second string is (T_1+w_2);

hence $$T_2=\tfrac{1}{2}(T_1+w_2)=\frac{W+w_1}{4}+\frac{w_2}{2}.$$

Similarly, $$T_3=\tfrac{1}{2}(T_2+w_3)=\frac{W+w_1}{8}+\frac{w_2}{4}+\frac{w_3}{2}.$$

Since the tension T_3 is equal to the external force (F) applied to the free end of the string,

$$F=\frac{W+w_1}{8}+\frac{w_2}{4}+\frac{w_3}{2}.$$

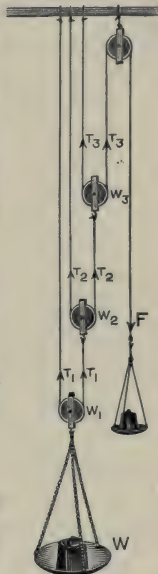


FIG. 60.—Expt. 69.

When the weights of the pulleys are neglected, this equation becomes $F=W/8$; or, the theoretical 'mechanical advantage' of the system is 8. But, in actual practice, the force must be sufficient (i) to lift the load, (ii) to lift the moving parts of the machine, and (iii) to overcome friction; hence the total force (F) must be greater than that represented by the equation $F=W/8$, and the true 'mechanical advantage' is less than 8.

The 'velocity ratio' is determined by the following reasoning: when W is raised 1 inch, the first string is shortened on each side by 1 inch; so that, for the string to remain taut, the wheel w_2 must rise 2 inches. For the same reason the wheel w_3 must rise 4 inches, and therefore the point of application of F will descend 8 inches. Hence the 'velocity ratio' is 8.

EXPT. 69.—**Principle of the pulley.** (i) Weigh three loose pulleys separately, and mark on each its weight. Fit up the system as shown in Fig. 60. Attach a load of about 1 lb., and adjust the force (F) until it is sufficient to raise W with *constant* velocity. Modify (F) until the weight W descends with constant velocity. Take the mean of these values of F. Compare this with the value of F deduced from the equation

$$F = \frac{W + w_1}{8} + \frac{w_2}{4} + \frac{w_3}{2}.$$

(ii) By measuring vertical distances of F and W from the floor, determine whether the *velocity ratio* agrees with that deduced from theory.

(iii) Determine the *mechanical advantage* of the machine, taking W simply as the load attached, and F as the force required to raise W with constant velocity.

(iv) Vary the load W, increasing it by 1 lb. at a time, and find in each case the force required to raise the load *with constant velocity*. From each observation calculate the *efficiency*, and also the *mechanical advantage*, of the machine.

Plot on squared paper the two curves (*a*) Load and Force, and (*b*) Load and Efficiency.

Record your observations in the following manner:

(i) $w_1 = 1{\cdot}38$ oz. Load (W) = 37 oz.
$w_2 = 1{\cdot}53$ oz.
$w_3 = 1{\cdot}47$ oz.
Force required (moving down) = 6·24 oz.
Force required (moving up) = 5·54 oz.
Average force required = 5·89 oz.

By theory, $F = \frac{W + w_1}{8} + \frac{w_2}{4} + \frac{w_3}{2} = \underline{5{\cdot}92}$ oz.

(ii) Distance traversed by F = 41·97 in.
Distance traversed by W = 5·2 in.
Velocity Ratio = 41·97/5·2 = 8·07.

(iii) **Mechanical advantage** = 37/6·24 = 5·93.

(iv) **Efficiency.**

Load (W).	Force (F).	Efficiency = $\frac{W}{F \times \text{Vel. Ratio}}$.	Mechanical Advantage = W/F.
53 oz.	8·34 oz.	$\frac{53}{66·72}$ = 0·79	6·36
69 ,,	10·49 ,,	$\frac{69}{83·92}$ = 0·82	6·58
85 ,,	12·64 ,,	$\frac{85}{101·1}$ = 0·84	6·73
101 ,,	14·74 ,,	$\frac{101}{117·9}$ = 0·86	6·86
117 ,,	16·89 ,,	$\frac{117}{135·1}$ = 0·87	6·93
133 ,,	18·94 ,,	$\frac{133}{151·5}$ = 0·88	7·02

Fig. 61 represents the curves drawn from the above data.

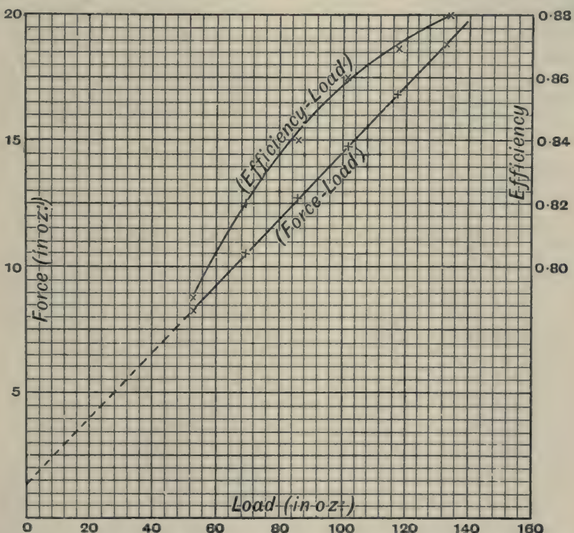


Fig. 61.—A graph of data obtained with the system of pulleys shown in Fig. 60.

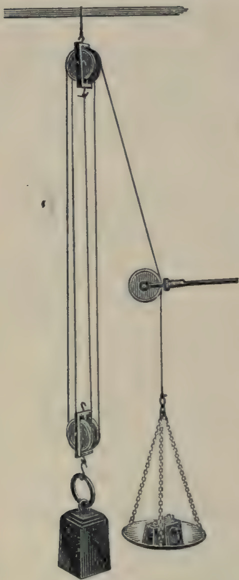


FIG. 62.—Expt. 70.

Two-sheave pulley blocks.—Fig. 62 represents a form of pulleys which is frequently used in practice; it consists of two blocks, each containing two or more independent sheaves, or wheels. One continuous cord is used, one end of the cord being permanently attached to the upper block, and the other end to the force applied. Since the tension of the cord is the same throughout its length, the total load (including the weight of the lower pulley block) is equal to *four* times the force applied when a two-sheave pulley block is used. Also, the velocity ratio is 4.

EXPT. 70.—**Practical pulley blocks.** With a pair of two-sheave pulley blocks, find (i) the velocity ratio, (ii) the mechanical advantage, and (iii) the efficiency when the load varies from 4 lb. to 10 lb.

HEAT.

CHAPTER IX.

THE FIXED POINTS OF THERMOMETERS. EXPANSION.

Fixed points.—The temperatures universally used in order to attach scales to mercury thermometers, are (i) the temperature of melting ice, and (ii) the temperature of steam from water boiling under normal atmospheric pressure. These temperatures, when marked on the stem of a thermometer, indicate the so-called **fixed points** of the thermometer. It must be remembered that the temperature of steam depends very much upon the pressure of the air at the time of the experiment; on the other hand, the temperature at which pure ice melts is, for all practical purposes, independent of variation in the atmospheric pressure. Cheap thermometers are liable to have appreciable scale errors; it is necessary therefore to verify their accuracy by determining the errors (if any) of the fixed points. If intended for any accurate work, it is necessary also to compare the readings at intermediate temperatures with those of a standard thermometer.

EXPT. 71.—**The lower fixed point.** Note the numbers, or marks, attached to a Centigrade thermometer and a Fahrenheit thermometer. Assuming that a solid block of ice is provided, wash it well under the tap, to remove all dirt. Wrap it in clean canvas, and break it into

small fragments. Fill a large glass funnel (Fig. 63) with the ice, and pour over it some distilled water. It is desirable to have a short piece of rubber tubing, closed with a clip, attached to the stem of the funnel. Close the clip and add more distilled water until the ice is thoroughly wetted.

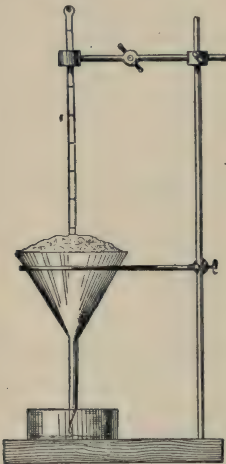


FIG. 63.—Verifying the lower fixed point of a thermometer.

Immerse the thermometers in the ice so that the lower fixed points are just visible; after remaining for several minutes, read the thermometers carefully to at least one-fifth part of a scale division, and note the readings. Remember that if the reading is *below* 0°C. (or 32° Fah.) the **correction** will be *positive*; if the reading is *above* 0° C (or 32° Fah.) the correction will be *negative*.

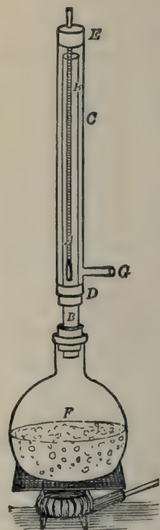


FIG. 64.—Verifying the higher fixed point of a thermometer.

EXPT. 72.—**The higher fixed point.** Fit up a simple form of hypsometer (Fig. 64). The steam passes up the inner tube B, descends down C and escapes at G. The thermometer, which is suspended from the cork E, should have a narrow rubber ring round its stem, in order to prevent it from falling, and placed so that the higher fixed point is just visible. *The bulb of the thermometer must not touch the surface of the water.*

Allow the thermometer to remain in the steam for several minutes, take the reading, and note also the reading of the barometer.

It may be assumed that the temperature of dry steam is increased, or diminished, by $0^\circ{\cdot}037$ C. for each millimetre change in the barometer height from 760 mm. With this datum calculate the true temperature of steam at the time of the experiment. Enter your results thus :

Height of mercury barometer,......
Calculated temperature of steam,......

	Centigrade thermometer.		Fahrenheit thermometer.	
	Error.	Correction.	Error.	Correction.
Lower fixed point				
Upper „ „				

Assuming that the bore of the tube is uniform, the amount of error at intermediate points will change at a

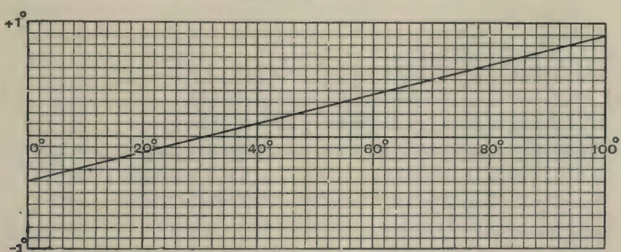


FIG. 65.—Corrections for a thermometer having incorrect fixed points.

uniform rate. Thus, suppose that the *corrections* for a Centigrade thermometer are $-0^\circ{\cdot}4$ and $+0^\circ{\cdot}9$ at the lower and higher fixed points respectively ; when these are marked on squared paper (Fig. 65) the straight line joining the two points will indicate the corrections at intermediate temperatures.

Linear Expansion of Solids.

Coefficient of linear expansion.—This coefficient is defined as **the increase in length experienced by a rod of the substance of unit length, at 0° C., when its temperature is raised through 1° C.**

When L_T, L_t and L_0 are the lengths of a rod or tube at temperatures T° C., t° C. and 0° C. respectively, and α is the coefficient of linear expansion, then

$$L_T = L_0(1 + \alpha T),$$

and
$$L_t = L_0(1 + \alpha t).$$

Hence $$L_T/L_t = (1 + \alpha T)/(1 + \alpha t) = 1 + \alpha(T - t) - \ldots.$$

(The higher terms involve α^2, and they may be neglected because α is a very small quantity.) Therefore

$$\alpha = (L_T - L_t)/L_t(T - t),$$

or, expressed in words,

$$\text{Coefficient, } \alpha, = \frac{\text{change in length}}{\text{length at lower temp.} \times \text{change in temp.}}.$$

The several different methods of measuring the coefficient differ from each other chiefly in the procedure adopted for measuring the small increase in length. In all cases, one end of the rod or tube is fixed, and the movement of the other end (or of some distinctive mark near this end) is measured by means of either a spherometer, or a micrometer screw-gauge, or by means of a microscope in the eye-piece of which a transparent scale is supported. In simpler forms of apparatus the small elongation is multiplied by means of a lever arrangement.

Expt. 73.—**Linear expansion (Method 1).** In Fig. 66. a metal rod about 30 cm. long is supported inside a metal steam jacket AB by means of a cork at its upper end. The lower end of the rod rests firmly on a glass plate supported on the base of the wooden stand. The stand also carries a disc of plate glass D horizontally at

the same level as the upper end of the metal rod. A circular hole is cut through the plate glass at its centre, and a small **V**-shaped depression is punched in the upper end of the rod.

Note the temperature (t) of the room. Measure accurately the length (L_t) of the rod. Fit up the steam jacket and clamp it *firmly* in position. See that the lower end of the rod touches the glass plate. Place a spherometer on the upper glass disc, screw down the centre leg until it just rests in the depression in the top of the rod, note the reading of the spherometer,

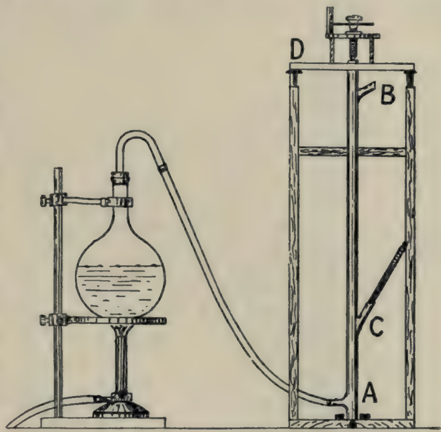


FIG. 66.—Coefficient of expansion of a metal rod.

and remove it until the next reading is to be taken. Pass steam through the steam jacket for five minutes ; during this period read the barometer and calculate the temperature (T) of the steam. Replace the spherometer and unscrew the centre leg until it again just touches the depression in the rod ; note the reading of the spherometer and, knowing the value of each division on the disc of the spherometer, calculate the change in length of the rod. From the observations calculate the coefficient of linear expansion of the metal.

If rods of other metals are available, their coefficients also should be determined.

EXPT. 74.—**Linear expansion (Method 2).** In Fig. 67 a metal tube AB (about 1 metre long and 2·5 cm. diameter) is fitted so that tap-water or steam can be passed through it. Near the end A a small **V**-shaped depression is punched in the surface of the tube. This end is firmly fixed by means of a wooden block, with circular hole, which is clamped to the table. A screw

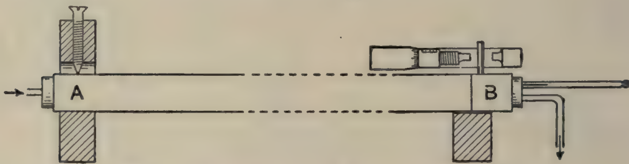


FIG. 67.—Coefficient of expansiou of a metal tube.

passes vertically down through the block, and the tube is placed so that the screw-point rests in the **V**-depression. Two methods of measuring the expansion are shown near the end B. In one of these, a short piece of thick steel knitting-needle is hammered tightly into a hole drilled through the top of the tube, and the movement of the needle is measured by means of a micrometer screw-gauge clamped *firmly* in a horizontal position.

The most sensitive method of determining the moment of contact between the gauge and the needle is obtained by connecting two copper wires respectively to the tube and to the gauge, and joining the free ends of the wires to an electric circuit containing a voltaic cell and a galvanoscope; by this means a movement of 0·005 millimetre is readily detected. The other method of measuring the expansion is to make a fine straight vertical scratch on the surface of the tube, by means of a stout needle; this scratch is focussed in a rigidly fixed microscope, with arbitrary scale in its eye-piece. The cork at the end B carries the thermometer and the exit tube.

Procedure.—Adjust the cork at the end B so that the exit-tube is uppermost: this facilitates the escape of air from the tube. Connect the end A with the tap, and allow water to flow through for several minutes. When the temperature of the tap water remains quite constant, note it (t), and measure the length (L_t) of the tube from the **V**-depression to the point under observation (either to the middle of the needle or to the scratch).

Take the reading of the screw-gauge (or of the microscope). Disconnect from the tap, allow the water to run out of the tube, by *gently* removing the cork at the end B. Push the thermometer into the cork until the 96° mark is just visible, and replace the cork with the exit tube in its lowest position. Connect the tube to a steam supply, and allow the steam to pass through for several minutes. It is well to wrap cotton wool, or flannel, round the tube so as to minimise the condensation. Note the reading of the screw-gauge (or of the microscope), and the temperature (T) of the steam. Calculate the coefficient of expansion of the metal.

Enter your observations thus:

Metal used: Brass.
Lower temp. (t) = 11°·7 C.
Length of tube (from fixed end to centre of needle),

$$L_t = 102{\cdot}04 \text{ cm.}$$

Micrometer screw-gauge reading, at t°C. = 6·16 mm.
Higher temp. (T) = 99°·9 C.
Micrometer reading at T°C. = 4·445 mm.
Expansion = 6·16 − 4·445 = 1·715 mm.

Hence, $$\alpha = \frac{0{\cdot}1715}{102{\cdot}04 \times (99{\cdot}9 - 11{\cdot}7)} = 0{\cdot}0000190.$$

EXPANSION OF LIQUIDS.

Apparent and real coefficients of expansion of a liquid.—When the temperature of a liquid contained in a vessel is raised, the expansion of the liquid observed is *less* than the amount by which it actually expands, owing to the fact that the vessel itself also expands, and its capacity therefore slightly increases. The amount by which one c.c. of the liquid appears to expand when warmed through 1° C., neglecting that the vessel also expands, is termed the **apparent** coefficient of expansion. To obtain the **real** coefficient of expansion it is necessary to add the cubical coefficient of expansion of the material of which the vessel is made; hence, if a glass vessel is used,

$$\left.\begin{array}{c}\textbf{Real}\text{ coefft. of}\\ \text{the liquid}\end{array}\right\} = \left.\begin{array}{c}\textbf{Apparent}\text{ coefft.}\\ \text{of the liquid}\end{array}\right\} + \begin{array}{c}\text{Cubical coefft. of}\\ \text{expansion of glass.}\end{array}$$

If β is the apparent coefficient of expansion of a liquid, V_t and V_T the volumes occupied by the same mass of liquid at temperature $t°$ C. and T° C., then

$$\beta = \frac{V_T - V_t}{V_t(T - t)},$$

or

$$V_T = V_t\{1 + \beta(T - t)\}. \qquad \text{(i)}$$

Since the density of a liquid is *inversely* proportional to the volume occupied by the same mass of liquid, equation (i) may be written

$$D_t = D_T\{1 + \beta(T - t)\}, \qquad \text{(ii)}$$

where D_t and D_T are the densities at $t°$ C. and T° C.

EXPT. 75.—**Apparent coefficient of expansion**. Fig. 68 represents a thin glass tube (about 30 cm. × 0·3 cm. bore) partly filled with the liquid. The tube is firmly attached to a thermometer. Place the arrangement in cold water, contained in a deep cylinder and, after keeping the temperature constant for 5 minutes, observe the degree on the thermometer scale which is level with the

top of the liquid column. Transfer the tube to a cylinder of warm water, keep the temperature constant for several minutes and again take the reading of the top of the liquid.

Remove the tube and, with a scale, measure the distance from the bottom of the tube to the points on the thermometer scale previously observed. If the bore is uniform the volume of liquid may be taken as proportional to the *length* of the column. Calculate the apparent coefficient.

FIG. 68.

EXPT. 76.—**Alternative method.** Find the weight (w_1) of a clean, dry "specific gravity" bottle (Fig. 40). Fill the bottle with the given liquid, and suspend it for 5 minutes immersed to the neck in a water bath at 18°-20° C. Keep the water stirred, and its temperature constant. Note this temperature (t). Remove the bottle, dry it in a folded cold duster, and weigh it (w_2). Re-suspend it in a water-bath at 50°-60° C., and proceed as before. Note the temperature (T), allow the bottle to cool, and find its weight (w_3).

Assuming that the capacity of the bottle is constant, the density of the liquid is proportional to the weight of the liquid contained in the bottle; hence, from equation (ii), p. 104,

$$(w_2 - w_1) = (w_3 - w_1)\{1 + \beta(T - t)\},$$

or

$$\beta = \frac{w_2 - w_3}{(w_3 - w_1)(T - t)}.$$

Expressed in words, the apparent coefficient

$$= \frac{\text{wt. of expelled liquid}}{\text{wt. of liquid at } \textit{higher} \text{ temp.}} \times \text{change of temp.}$$

The coefficient of linear expansion of glass may be taken as 0·000008; the *real* coefficient of expansion of the liquid is calculated, therefore, by adding 0·000024 to the value obtained for β.

The *real* coefficient of expansion may be measured directly by containing the liquid in a vessel arranged so that its capacity is constant at all temperatures. This condition is obtained by using a glass vessel which is partly filled (to one-seventh part of its total capacity) with mercury.

The anomalous expansion of water.—At low temperatures the coefficient of expansion of water is so small that any peculiarities are masked by the change in volume of the vessel containing the water. A vessel of constant capacity is obtained if about one-seventh part of the total capacity of a glass vessel is occupied by mercury (Fig. 69). A suitable apparatus consists of a cylindrical bulb, about 10 cm. long and 1·5 cm. diameter, terminating in a capillary tube having a bore of about 0·5 mm.

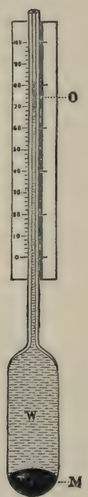


FIG. 69.—The temperature of water at its maximum density.

EXPT. 77.—**Temperature of maximum density of water.** Weigh the empty tube (Fig. 69), fill it with mercury, and again weigh. Invert the tube, alternately heat and cool it until six-sevenths of the mercury has been expelled. Again fix the tube in an upright position, and fill it with *well-boiled* distilled water. Introduce a short length of oil (O), to prevent evaporation and absorption of air; and fix a paper millimetre scale to the capillary tube.

Support this apparatus in a wide test-tube containing mercury, so as to secure uniformity of temperature.

Place a thermometer in the mercury, and support the wide tube containing it and the apparatus in a beaker of cold water. Notice the position of the top of the liquid in the tube, and read the temperature shown by the thermometer. Add ice to the water, and as the temperature falls notice the level of the liquid in the tube for every degree down to 1° or 2° C.

Then let the water in the beaker gradually rise in temperature, adding a little warm water, if necessary, and again observe the positions at the same temperatures as before. The mean of the two positions observed for each temperature should be taken as the true reading for that particular temperature.

FIG. 70.—Graphic representation of changes in volume of water near the freezing point.

Construct a curve like that shown in Fig. 70 to represent your observations of the changes of volume of water at temperatures near the freezing point.

At what temperature has the water in the apparatus the least volume, and therefore the maximum density?

EXPANSION OF GASES.

Charles's law.—The law may be expressed thus: **Equal volumes of all gases under the same constant pressure, expand by the same fraction of the volume at 0° C. for every degree rise in temperature.**

Or, if a volume V_0 of a gas, measured at 0° C., expands to V_T when warmed to T° C., then

$$\frac{V_T - V_0}{V_0 \times T} = \alpha,$$

where α is a constant, which is the **coefficient of expansion** under constant pressure.

Hence, $V_T - V_0 = \alpha V_0 T$,

or $V_T = V_0(1 + \alpha T)$.

As, in experimental work, it is not always convenient to measure the volume at 0° C., it is more usual to measure the volumes at two other temperatures t° C. and T° C.; then

$$V_T = V_0(1 + \alpha T) \quad \text{.......................(i)}$$

and

$$V_t = V_0(1 + \alpha t). \quad \text{.......................(ii)}$$

Hence $V_T/V_t = (1 + \alpha T)/(1 + \alpha t)$,

or $V_T(1 + \alpha t) = V_t(1 + \alpha T)$,

or

$$\alpha = (V_T - V_t)/(V_t T - V_T t). \quad \text{.........(iii)}$$

On comparing this expression with the corresponding expression (p. 100) for the coefficient of linear expansion of a solid, it will be noticed that the two expressions would be identical if, in the denominator of equation (iii), V_t were substituted for V_T. In the case of solids the expansion is so small that L_t may be substituted for L_T without introducing appreciable error in the result; whereas, in gases, the expansion is much greater, and V_t cannot therefore be substituted for V_T.

EXPT. 78.—**Expansion of gas at constant pressure.** Dry thoroughly a 500 c.c. flask, and fit it with a one-holed *rubber* stopper through which passes a short glass tube with rubber tubing and clip attached. Fix the flask in an open metal vessel (as shown in Fig. 71) sufficiently large for the flask to be immersed completely in boiling water. The flask can be held immersed by means of an inverted stout glass jar held in a clamp at its upper end.

In order to prevent steam from entering the flask it is well to attach a long glass tube to the open end of the rubber tubing. Keep the clip open, and gradually heat the water to boiling. Let the water boil for about five minutes, and close the clip. Note the temperature of the water. Remove the flask, immerse it in a bath of cold water with its neck downwards. Open the clip, and keep the flask moving in the water so that the remaining air acquires the same temperature as the water.

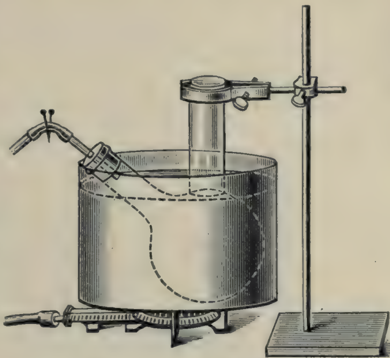


FIG. 71.—Determination of the coefficient of expansion of a gas at constant pressure.

Lower the flask till the level of the water is the same inside and outside; then close the clip. Note the temperature of the cold water. Remove the flask, and measure the volume of water which has entered the flask. Find also the volume of the whole flask up to the lower surface of the stopper.

A method of calculation is indicated by the following example of an experiment:

Volume of whole flask - -	- 340 c.c.
„ water drawn into flask	- 77 c.c.
Temperature of boiling water	- 98° C.
„ cold „	- 15° C.

Hence $(340-77)$ c.c. at 15° C. expand to 340 c.c. when warmed to 98° C.

Therefore, $V_t = 263$ c.c.; $t = 15°$.
$V_T = 340$ c.c.; $T = 98°$.

or $$\alpha = \frac{77}{(263 \times 98) - (340 \times 15)} = \frac{77}{20678} = \frac{1}{268 \cdot 5}.$$

Plot the readings on squared paper (Fig. 72); and, assuming that the rate of contraction, due to cooling, is uniform, determine at what temperature the volume

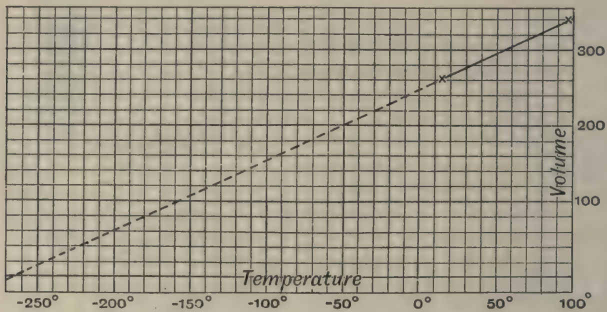


FIG. 72.—The contraction of a gas due to cooling.

will theoretically be reduced to zero. This temperature is defined as the **absolute zero of temperature**.

EXPT. 79.—**Alternative method.** Obtain a piece of thermometer tubing, about 60 cm. long and 1 mm. bore. Thoroughly dry the inside by drawing through it a stream of dry air, and at the same time heating it from the outside. Draw into the tube a thread of *clean* mercury, about 2 cm. long, and adjust its position about 20 cm. from one end. Hold the tube horizontally, and with a blow-pipe flame seal off the tube at a point about 10 cm. from the other end. The tube now contains a column of dry air enclosed by the mercury thread. If the air column is inconveniently long, it

may be adjusted by holding the tube vertically and inserting a length of clean steel wire until its end projects beyond the mercury thread. Fasten a thermometer to the tube by means of rubber bands.

Place the apparatus in a tall cylinder of ice-cold water (Fig. 73); add ice occasionally to keep the temperature constant for about five minutes, keeping the water well stirred. Hold the tube vertically and read the position of the top of the air column relatively to the thermometer scale. Note also the temperature. Empty out the cold water and replace it with hot water. Keep the temperature constant for 5 minutes and repeat the readings.

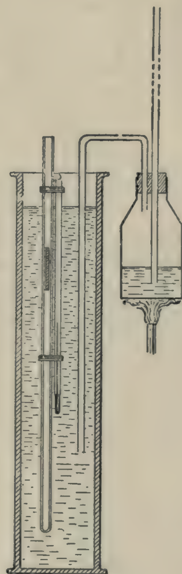


FIG. 73.—Coefficient of expansion of a gas.

Fig. 73 suggests a convenient method of keeping the temperature constant by occasionally passing into the water some steam from a boiling can at the side of the apparatus. Remove the tube and thermometer and measure with a scale the lengths of the air column at the two observed temperatures. Assuming that the bore is constant, the volume of the air is proportional to the measured lengths. Calculate the coefficient of expansion of air.

Coefficient of increase of pressure of a gas, at constant volume.—When a gas is heated while its volume is kept constant, its pressure increases according to the same law as that which holds good for the increase of volume when the pressure is kept constant. Hence, if P_0, P_t, and P_T

represent the pressures at the temperatures 0° C., t° C., and T° C. respectively,

$$P_t = P_0(1+\alpha t) \quad \text{and} \quad P_T = P_0(1+\alpha T).$$

Hence, $$\frac{P_T}{P_t} = \frac{1+\alpha T}{1+\alpha t}.$$

EXPT. 80.—**The pressure coefficient of a gas.** Fig. 74 represents a suitable arrangement of apparatus. A glass bulb A containing dry air is connected by a horizontal capillary tube to the tubes EC and BD which are fitted up in a manner similar to that required for demonstrating Boyle's Law.

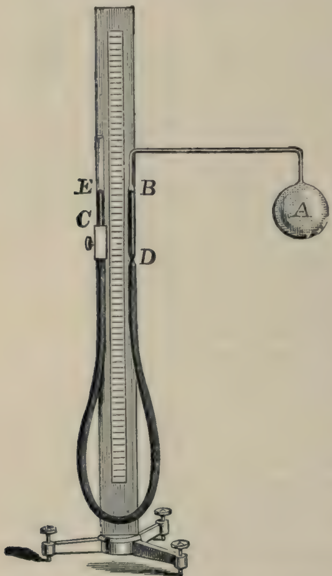


FIG. 74.—Coefficient of increase of pressure of a gas at constant volume.

Surround the bulb A with water contained in a metal vessel; keep the temperature constant for several minutes, then adjust the position of the tube EC so that the mercury surface at B coincides with a horizontal line previously marked on the outside of the tube. If h be the difference of level of the mercury surfaces at E and B, and if H be the height of the barometer, the total pressure (P_1) is equal to ($H \pm h$), the sign depending upon whether the mercury surface at E is above or below the surface at B. Note also the temperature (t) of the water bath. Now raise the temperature of the water bath to T, and repeat the previous observations. Calculate the value of α by means of the above equation.

CHAPTER X.

MEASUREMENT OF HEAT. SPECIFIC HEAT. LATENT HEAT.

Measurement of heat.—Quantity of heat is measured in terms of the **calorie** (or **therm**), which is defined as **the amount of heat required to raise the temperature of one gram of water through 1° C.** : the same quantity of heat is given out when one gram of water cools through 1° C. Hence, the quantity of heat required to raise the temperature of m grams of water through t° C. is $(m \times t)$ calories. Water is the only substance for which this statement is true ; the same weight of any other substance, in cooling through the same range of temperature, will give out less heat ; and the quantity depends upon the substance.

EXPT. 81.[1]—**The quantity of heat given out on cooling depends upon the substance.** Select two thermometers the readings of which mutually agree. Half fill a litre beaker with water, and heat it to boiling. Weigh out equal weights (50-70 gm.) of different substances, *e.g.* water, cast-iron nails, lead sheet, broken glass, etc., and put these into separate boiling tubes. Close each tube with a plug of cotton wool, and place the tubes in the beaker of boiling water (Fig. 75). Insert one of the thermometers in one of the tubes—say, the tube containing

[1] In all experiments in which two or more thermometers are to be used, it is always advisable to test them beforehand, to see whether the readings agree. This may be done by immersing them for a time in the same beaker of water, the water being kept stirred. The readings should not differ by more than 0°·2 C. This is a very necessary precaution when cheap thermometers are to be used.

the iron—with its bulb in contact with the substance.

Weigh a metal vessel, *e.g.* a **calorimeter,** large enough to hold 200 grams of water. Pour into it about 100 c.c. of water, and again weigh. Wrap cotton wool round the vessel, and place the vessel inside a larger one which will serve to support it firmly. Place the other thermometer in the water. Read both thermometers every minute for several minutes; and *when the readings of both are steady*, note them. Quickly remove the cotton wool and thermometer from the boiling tube, and transfer the contents to the vessel of water. Gently stir the water, and watch the thermometer; when the reading is steady for a few seconds, note the temperature. Empty out the contents of the vessel, dry it, and repeat the observations with the other substances, *using in each case the same weight of water in the calorimeter.*

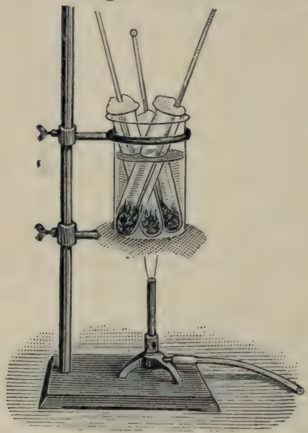


FIG. 75.—Method of heating substances, for measuring their heat-capacity.

Record your observations thus:

weight of vessel, 18·61 grams

" " *and water,* 100 grams.

Substance.	Weight.	Fall of temp. ° C.	Rise of temp. of water. ° C.	Calories given up to water.	Calories given up in cooling 1° C.
(i) Iron	70 gm.	97 − 23·1 = 73·9	23·1 − 15·8 = 7·3	81·4 × 7·3 = 586·1	586·1/73·9 = 7·93
(ii) Lead	70 gm.	97 − 18 = 79	18 − 16 = 2	81·4 × 2 = 162·8	162·8/79 = 2·06

In this experiment we have disregarded the fact that a certain quantity of the heat given up by the hot substance has been used in warming the containing vessel; in absorbing this heat the vessel therefore is *equivalent* to a small additional quantity of water, and the weight of water to which the vessel is equivalent is termed its **water equivalent.** In accurate experiments this must be taken into consideration, and it may be determined by experiment—although it is more accurate to *calculate* the value of the equivalent by the method described in the next section.

EXPT. 82.—**The water equivalent of a calorimeter.** Weigh a calorimeter, and wrap cotton wool round it sufficient to allow it to rest steadily in a larger vessel. Place inside the calorimeter a thermometer (with scale divided to $0^\circ\cdot 2$ C.); leave it there for five minutes. Nearly fill a 4 oz. flask with water, and heat it to nearly 40° C.; close the neck with the thumb, and hold the flask inverted, shaking it meanwhile so as to warm the whole of the flask to a uniform temperature. Note the temperature (t) of the calorimeter, and the temperature (T) of the warm water. Quickly pour into the calorimeter enough of the warm water to cover the bulb of the thermometer. Use this thermometer as a stirrer, watch the mercury thread, and note the final temperature (θ). Remove the thermometer, and find the weight (w) of water added by again weighing the calorimeter. Enter your observations thus:

Weight of calorimeter = ...
Initial temp. of calorimeter (t) = ...
" " hot water (T) = ...
Final temp. (θ) = ...
Weight of calorimeter and water = ...
Weight of water added (w) = ...
Heat given up by warm water $= w(T-\theta) =$ ... calories.

Hence, heat required to warm calorimeter through 1° C.

$$=\frac{w(T-\theta)}{(\theta-t)}=\ldots\ldots \text{ calories;}$$

and, *water equivalent* of calorimeter = ... grams.

In this experiment no allowance has been made for the heat absorbed by the immersed part of the thermometer; this has a definite water-equivalent, but in experiments where great accuracy is not required, the error introduced by disregarding the effect of the thermometer may be neglected.

Specific heat.—The results of Expt. 81 will show that the quantity of heat given up by a mass of iron in cooling through 1° C. is only about one-eighth part of the quantity given up by the same mass of water. This ratio is termed the **specific heat** of the iron. The specific heat of a substance may be defined therefore as **the ratio of the quantity of heat given up by a known weight of the substance in cooling through 1° C., to the quantity given up by the same weight of water in cooling through 1° C.** The ratio is the same, of course, for the quantities of heat required to warm equal weights of the substances through 1° C. Hence, we may write

$$\textbf{Specific heat}=\frac{\textit{heat given up by any weight of the substance in cooling } 1^\circ\textit{ C.}}{\textit{heat given up by the same weight of water in cooling } 1^\circ\textit{ C.}}.$$

If the weight taken happens to be 1 gram, the denominator becomes unity, and the specific heat of the substance is then *numerically* equal to the numerator. The student should avoid the error of defining the specific heat simply as "the heat given up by 1 gram of the substance in cooling through 1° C.": such a definition would imply that specific heat is a quantity of heat expressed in calories; whereas it is really a numerical ratio only.[1]

If one gram of a substance, of specific heat s, is cooled through 1° C., the quantity of heat given out is s calorie; if w grams of the same substance are taken, the heat given out

[1] The distinction may be compared to that which is recognised between *density* and *specific gravity*.

is increased to *ws* calories. Hence this weight of the substance is equivalent to *ws* grams of water: this quantity is the **water-equivalent** of the substance. In the case of a calorimeter, the rule for calculating its water-equivalent is

water-equivalent = weight × specific heat of the metal.

Assuming that the specific heat is known, this rule gives a more trustworthy value of the water-equivalent than the method described in Expt. 82.

EXPT. 83.—**Specific heat of a solid.** Weigh out about 50-70 grams of the substance (other than copper); place it in a large test-tube immersed in boiling water, as shown in Fig. 75. When the substance is in large fragments, such as can be tied into a bundle by cotton thread, it is more accurate to heat it in a *steam-jacket* (Fig. 76). When the temperature of the central chamber is quite steady, the calorimeter is quickly put into a position immediately below the steam-jacket, the bottom lid is swung open, and the solid is *expeditiously* lowered into the calorimeter, which is then removed to a distance and the experiment continued.

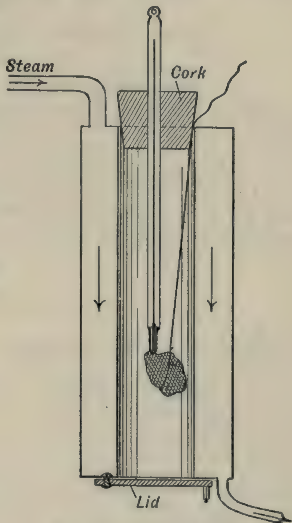


FIG. 76.—A steam-jacket.

While the substance is being heated, weigh a copper calorimeter; and, by reference to a table of specific heats, calculate its water-equivalent: a suitable size of calorimeter is 3 in. deep by 2 in. diameter. Pour in enough cold water to fill it to about two-thirds. Again

weigh it. Wrap cotton wool round it, as in previous experiments. Note (i) the temperature (t) of the cold water, and (ii) the temperature (T) of the hot substance. Quickly transfer the hot substance to the calorimeter, stir the water, and note the final temperature (θ) of the water.

Record your observations thus :

Weight of substance (W) = ... gm.
" calorimeter = ... gm.
Water-equivalent of calorimeter (e) = ... gm.
Weight of calorimeter and water = ... gm.
Weight of water (w) = ... gm.
Initial temp. of the cold water (t) = ...° C.
" " hot substance (T) = ...° C.
Final temperature of mixture (θ) = ...° C.
Heat absorbed by water and calorimeter

$$= (w + e)(\theta - t) = \text{......calories.}$$

Hence, heat given out by W grams of substance in cooling $(T - \theta)$ degrees is $(w + e)(\theta - t)$ calories ; and heat given out by 1 gram of substance in cooling 1 degree is

$$\frac{(w + e)(\theta - t)}{W(T - \theta)} \text{ calories.}$$

This is numerically equal to the *specific heat* of the substance.

In this experiment no correction has been made for the heat lost from the calorimeter during the interval between the immersion of the hot substance and the reading of the final temperature: during this period the temperature of the water is always higher than that of the surrounding air, and heat therefore is lost by radiation. Reference must be made to a more advanced text-book for the method of finding the heat so lost. One method of minimising this error is to repeat the experiment described above, taking the same weight of water as before, but cooled sufficiently so that the average of the initial and final temperatures of the water is equal to the temperature of the room.

When the solid is soluble in water, a liquid must be used in which the solid is insoluble; and the specific heat of the liquid must either be determined previously, or obtained from reference tables.

EXPT. 84.—**Specific heat of a liquid.** The most satisfactory method is to introduce into an observed weight of the liquid an observed weight of a solid, the specific heat of which is known. Turpentine, glycerin or petroleum are convenient liquids, and iron (specific heat, 0·112) is a suitable solid.

Weigh accurately 30-40 grams of iron and heat it by the method described in the previous experiment. Weigh an empty calorimeter, pour in turpentine until it is at least half full, and again weigh. Proceed exactly as described in the previous experiment.

Using the same symbols, the heat given up by the hot iron is

$$\{W \times 0{\cdot}112 \times (T - \theta)\} \text{ calories.}$$

Part of this heat, viz. $e(\theta - t)$ calories, is used in warming the calorimeter. Hence

$$\{W \times 0{\cdot}112 \times (T - \theta)\} - e(\theta - t) \text{ calories}$$

will warm w grams of the liquid through $(\theta - t)$ degrees, and

$$\frac{\{W \times 0{\cdot}112 \times (T - \theta)\} - e(\theta - t)}{w(\theta - t)} \text{ calories}$$

will warm 1 gram of the liquid through 1° C.

This quantity of heat is numerically equal to the specific heat of the liquid.

Providing that no chemical action takes place between the liquid and water, hot water may be used instead of a hot solid. But as there is always some uncertainty as to the actual temperature of the water *when it reaches the calorimeter* the result is not so trustworthy.

The specific heat of a liquid may be determined also by *the method of cooling* (as described in Expt. 101, p. 140).

LATENT HEAT.

Latent Heat.—When a vessel containing a mixture of water and ice is heated over a gas flame, and kept well stirred, the temperature is found to remain constant (viz. 0° C.) *until all the ice has melted.* This demonstrates that heat is absorbed in changing solid ice into water, without raising the temperature. The quantity of heat required to melt one gram of ice at 0° C. is termed **the latent heat of fusion of ice** (or, **the latent heat of water**). A similar observation with boiling water demonstrates that heat is absorbed in converting boiling water into steam; and the quantity of heat required to convert one gram of boiling water into steam is termed **the latent heat of vaporisation of water** (or, **the latent heat of steam**).

EXPT. 85.—**Latent heat of fusion of ice.** Weigh a metal calorimeter. About half fill the calorimeter with water, previously warmed to about 35° C., and again weigh. Break some ice into small pieces, and place within the folds of some blotting-paper, in order to dry the ice, a quantity sufficient to weigh about one-fifth as much as the warm water. Stir the water with a thermometer, and when the temperature has fallen to about 30° C., note the exact temperature, and transfer the dry ice into the calorimeter. Keep the contents continually stirred until all the ice is melted, and at once note the temperature. In order to find the weight of ice added, take a final weighing of the calorimeter and its contents. Enter the observations thus:

Weight of calorimeter (w_1) = ... gm.
Water equivalent to calorimeter (e) = ... gm.
Weight of warm water (w_2) = ... gm.
" ice (w_3) = ... gm.
Initial temp. of water (T_1) = ...° C.
Final " " (T_2) = ...° C.
Latent heat of water = L heat units.

Heat gained by the ice

$$=\left.\begin{matrix}\text{heat required to}\\ \text{melt the ice}\end{matrix}\right\}+\left.\begin{matrix}\text{heat required to warm}\\ \text{melted ice up to } T_2^\circ\end{matrix}\right\}$$

$$=(w_3 \times L)+(w_3 \times T_2) \text{ calories.}$$

Heat lost by warm water and calorimeter

$$=(w_2+e)(T_1-T_2) \text{ calories.}$$

But, heat lost by warm water = heat gained by the ice.

Hence, $(w_2+e) \times (T_1-T_2)=(w_3 \times L)+(w_3 \times T_2)$,

or
$$L=\frac{(w_2+e)(T_1-T_2)-w_3T_2}{w_3}.$$

EXPT. 86.—**Latent heat of vaporisation of water.** Arrange a flask with the connections shown in Fig. 77; the short length of wider glass tubing is a trap to catch any condensed steam. While water in the flask is being heated to boiling, weigh a large copper calorimeter, add about 250 grams of water which has been previously cooled to about 10°C., and again weigh. After steam has been issuing from the glass tube for a few minutes, note the temperature of the cold water, place the calorimeter so that the end of the tube is well immersed in the water, and allow it to remain there

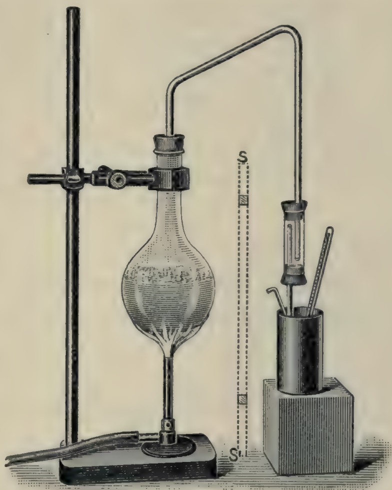


FIG. 77.—Determination of the latent heat of vaporisation of water.

until the temperature of the water has been raised to about 25° C. Remove the calorimeter, stir the water and quickly note its final temperature. Weigh the calorimeter again in order to find the weight of steam condensed. Enter your observations thus:

Weight of calorimeter (w) = ... gm.
Water equivalent of calorimeter (e) = ... gm.
Weight of cold water (W_1) = ... gm.
" steam (W_2) = ... gm.
Initial temp. of cold water (t) = ...° C.
Final " " (T) = ...° C.
Latent heat of steam = L calories.

Heat gained by cold water and calorimeter

$$= (W_1 + e)(T - t) \text{ cal.}$$

Heat lost by steam

$$= \left.\begin{matrix}\text{latent heat of} \\ W_2 \text{ gm. steam}\end{matrix}\right\} + \left.\begin{matrix}\text{heat given up by } W_2 \text{ gm. water} \\ \text{in cooling from } 100^\circ \text{ to } T^\circ\end{matrix}\right\}$$

$$= W_2L + W_2(100 - T) \text{ cal.}$$

But, heat lost by steam = heat gained by cold water, etc.; hence

$$W_2L + W_2(100 - T) = (W_1 + e)(T - t),$$

or

$$L = \frac{(W_1 + e)(T - t) - W_2(100 - T)}{W_2}.$$

Verify that the temperature of the steam is 100° C.; if it differs appreciably, make the necessary correction.

CHAPTER XI.

MELTING POINT AND BOILING POINT. VAPOUR PRESSURE. DEW POINT.

EXPT. 87.—**Melting point of paraffin wax.** With a blow-pipe flame draw out to capillary size a piece of thin-walled glass tubing. Cut off about 10 cm. of the capillary tube, warm it and dip one end into the wax, which has been previously melted in a test-tube immersed in boiling water. Remove the capillary tube and, when the wax in it has solidified, cut off this part of the tube and fuse the end in a small flame. With thread (or narrow rubber band cut from tubing) fasten the tube to the bulb of a thermometer; the tube should be sufficiently long for the open end to be above the surface of the water when the thermometer bulb is immersed (Fig. 78).

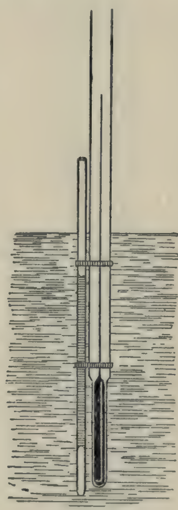


FIG. 78.

Place the thermometer in a beaker of cold water, and warm it *gradually* with a small flame. Keep the water constantly stirred, by using a separate stirrer, or the thermometer itself may be used for the purpose.

Carefully watch the wax, and note the temperature when it becomes transparent. Remove the flame, continue to stir the water, and note the temperature when the wax becomes opaque. The mean of the two temperatures observed is the melting point of the wax.

The melting points of beeswax, lard and butter may be determined by the same method.

In the case of substances the melting point of which is above 100° C., either sulphuric acid or a strong solution of calcium chloride must be used instead of water; it is advisable to support the beaker on a large sand-bath instead of the customary wire gauze. The melting point of sulphur would be determined in this manner.

EXPT. 88.—**Melting point (alternative method).** Determine the melting point of naphthalene, or of paraffin wax, by the following procedure: Fit a small test-tube (2 inches long) with a cork and thermometer (Fig. 79), and cut a small groove in the side of the cork so that it shall not fit air-tight. Melt some naphthalene in the tube. Clamp the thermometer in a vertical position with the test-tube within the inner vessel of a water jacket which serves to maintain uniformity of temperature of the surroundings and to avoid the effects of draughts. Any small metal vessel, weighted, and standing in a small pneumatic trough full of water, serves as an efficient water jacket.

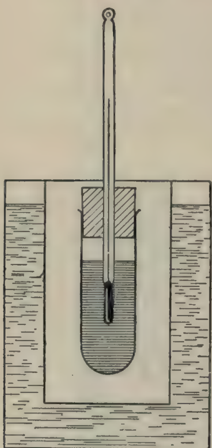


FIG. 79.—Melting point (by cooling curve).

Take readings of the temperature every half-minute until the substance has cooled to about 40° C. Plot

the readings on squared paper (Fig. 80). Notice that, in one part of the curve, the temperature remains more or less constant: this is due to the fact that the latent heat given up by the liquid on solidifying more or less counteracts the loss of heat by cooling.

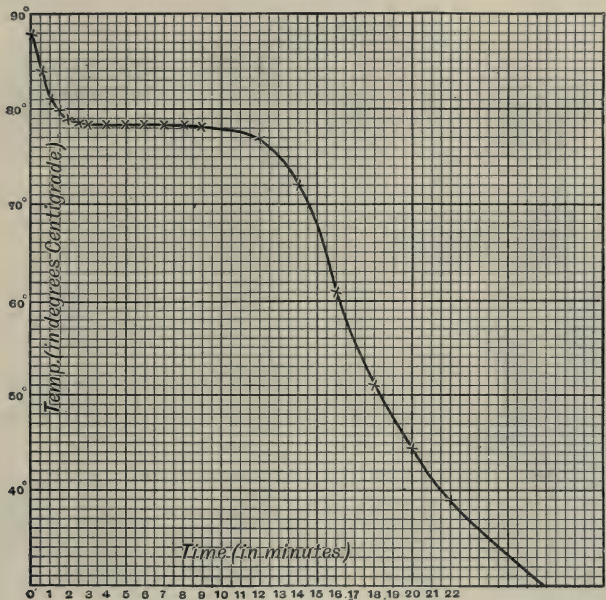


FIG. 80.—Cooling curve of melted naphthalene.

EXPT. 89.—**Boiling point of alcohol**. Fit a boiling-tube A (Fig. 81) with a cork through which pass a thermometer and a long glass tube C. Support A in a beaker B containing water. Pour alcohol into A, and add a few fragments of glass rod so as to ensure steady boiling. The tube C serves to condense the vapour of the alcohol, and it lessens the possibility of the vapour

taking fire. Heat the water in the beaker until the alcohol boils, and note the reading of the thermometer. At the same time read the height of the barometer.

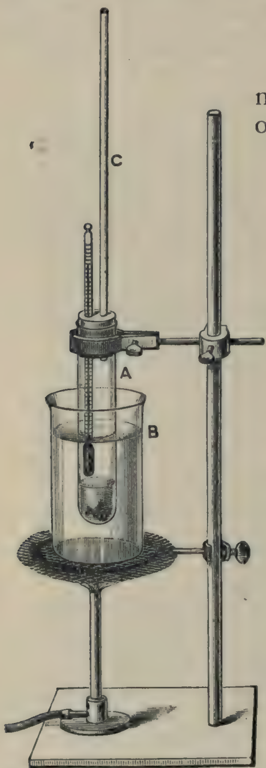


FIG. 81.—Determination of the boiling point of alcohol.

An alternative method of determining a boiling point is described on p. 129.

EXPT. 90.—**Effect of change of pressure on the boiling point of water.** Fit up the apparatus shown in Fig. 82, in which C is a glass tube bent twice at right angles, tapered slightly at the lower end, and cut off in a slanting direction. D is a narrow cylinder into which mercury may be poured, whereby the pressure is increased when steam is escaping through the mercury. The total pressure is obtained by adding the difference of level between α and β to the height of the barometer. By adding successive quantities of mercury obtain a series of simultaneous readings of boiling point and pressure.

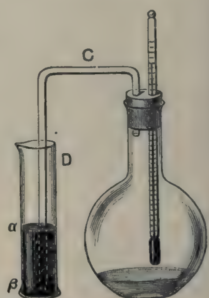


FIG. 82.—Arrangement for determining the effect of increased pressure upon the boiling point.

Obtain a similar series of readings under diminished pressure thus: leave a small quantity of mercury in D, and

turn down the Bunsen burner. The mercury now rises up the tube C, and diminishes the pressure. The pressure is obtained by subtracting from the barometer-reading the difference of mercury level in C and in D. More rapid boiling is ensured by wrapping blotting-paper wetted with cold water round the neck of the flask.

VAPOUR PRESSURE.

EXPT. 91.—**The vapour pressure of water at different temperatures.** Carefully fit up two barometers (Fig. 83) with their open ends in a shallow trough of mercury. Slide down them a two-holed rubber bung, and erect on the bung the wide glass tube, which is sufficiently long to extend beyond the tops of the barometers.

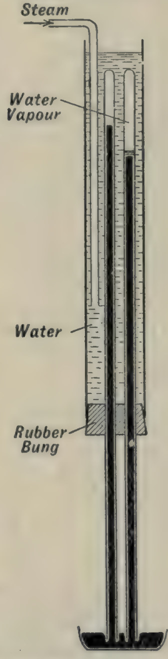


FIG. 83.—Measurement of vapour pressure.

Support the apparatus firmly by means of a retort stand and clamps so that the lower ends of the tubes do not rest on the bottom of the trough. Nearly fill the wide tube with cold tap water. Draw into a bent pipette (Fig. 84) some *well-boiled* distilled water, immerse the end below the mercury in the trough and, by gentle blowing, force a few drops of the water to leave the pipette; then, without removing the pipette, place the bent end immediately below one of the barometer tubes, and force a *few* drops out of the pipette; these will rise to the top of the mercury column and form a shallow

layer of water at the top, at the same moment the mercury column will fall through a short distance. *The difference between the heights of the two mercury columns represents the vapour pressure of water at the temperature of the surrounding water.*

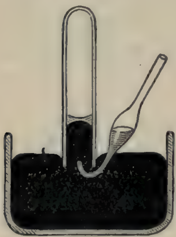


FIG. 84.

Stir the water thoroughly, note its temperature, and by means of a metre scale held vertically with its lower end resting on the bench, measure the difference between the heights of the mercury columns. One method of effectually stirring the water is to blow air, by the mouth, down a long narrow glass tube immersed in the water and reaching nearly to the bottom of the water-jacket.

Warm the water about 10° by passing steam into it from a boiler, such as is shown in Fig. 73 (p. 111); well stir the water, keep its temperature constant for at least five minutes, and repeat the previous readings. Continue this procedure at higher temperatures up to about 60° C. When necessary, water can be removed from the water-jacket by siphoning. Tabulate the observations thus:

Temperature.	Vapour pressure of water.

Plot the observations on squared paper, taking temperatures as abscissae, and vapour pressures as ordinates.

The same procedure may be adopted with other liquids, *e.g.* alcohol and ether (for temperatures not exceeding about 32° C.).

EXPT. 92.—**The vapour pressure of water at its boiling point.** In Fig. 85, A is a U-tube with limbs about 30 cm. long, and with one limb sealed. The latter limb is surrounded by a wide glass tube B closed at both ends with corks. The upper cork is fitted with a glass tube for the purpose of leading steam into B; the lower cork supports the U-tube, and it has also a side tube for the escape of condensed steam.

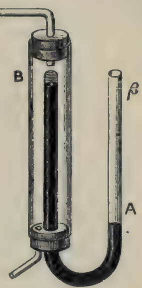


FIG. 85.—Experiment on the vapour pressure of a liquid at its boiling point.

Fit up the tube A as follows: fill it completely to the point β with clean dry mercury, close the open end with the thumb, tilt the tube so that the air bubble at β travels round to the closed end, and finally bring the bubble back to the open end. The walls of the tube should now be quite free from small air bubbles. Introduce two or three drops of well boiled distilled water into the tube so as to fill it completely; close the end with the thumb, and tilt the tube so that the water travels round to the closed end. Finally, withdraw mercury from the open end, by means of a narrow pipette, until the mercury surface is just above the bend. Pass steam through the steam-jacket B, and notice how the vapour pressure of the water depresses the mercury in contact with it *until the two mercury surfaces are at the same level.*

This experiment suggests an effective method of determining the boiling point of any liquid, and it is particularly useful when only a small quantity of the liquid is available.

EXPT. 93.—**Boiling point of a liquid (vapour pressure method).** Make a narrow glass U-tube similar to Fig. 86,

and with a closed limb at least 10 cm. long. Introduce mercury and alcohol, by the procedure explained in Expt. 92. Support the tube in a beaker of water, and support a thermometer in the water. Gradually warm the beaker, keeping the water well stirred, and note the temperature when the mercury surfaces in the two limbs of the U-tube are at the same level.

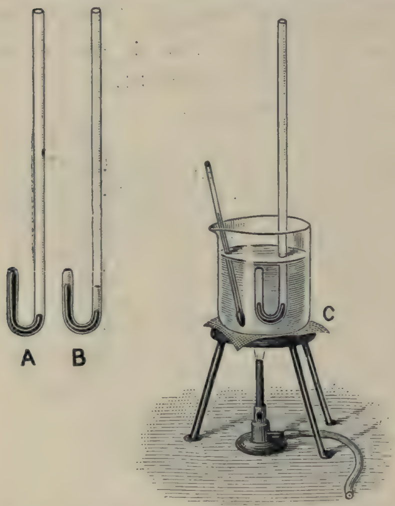


FIG. 86.—Boiling point of a liquid (vapour pressure method).

Other tubes should be fitted up for determining the boiling points of ether, benzene and chloroform.

DEW POINT AND RELATIVE HUMIDITY.

The **dew point** is the temperature to which the air must be cooled in order that the moisture present will suffice to saturate the air at that temperature.

The **relative humidity** of the air at any moment is the ratio of the weight of water vapour present (in a given volume of the air) to the weight of water vapour which would be present in the same volume if the air were saturated. Since the pressure of the vapour is proportional to the weight of vapour present, it is more usual to express the humidity thus:

$$\text{relative humidity} = \frac{\text{pressure actually exerted (\textit{i.e.} maxim. press. at dew-point)}}{\text{maxim. press. at the temp. of the room}}.$$

EXPT. 94.—**Dew point.** About half-fill the aluminium cup A (Fig. 87) with water. Suspend a thermometer B, graduated to $0^\circ \cdot 2$ C., in the water. Place a large sheet of glass in front of the apparatus so as to screen it from the warmth and breath of the observer. Add a *small* fragment of ice, and stir continually until the ice is melted. Add another small fragment of ice, and stir until melted. Continue this process until the deposition of dew upon the cup is observed; note the temperature of the water. Continue to stir the water, and note the temperature when the deposited dew disappears.

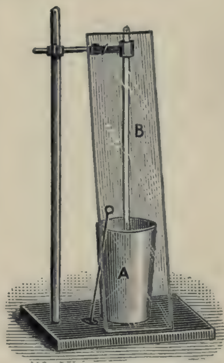


FIG. 87.—Aluminium cup hygrometer.

The average of these two temperatures is the *dew point.* Observe the temperature of the room, and, from the data of vapour pressures of water, given in Physical Tables (p. 236), calculate the relative humidity of the air in the room.

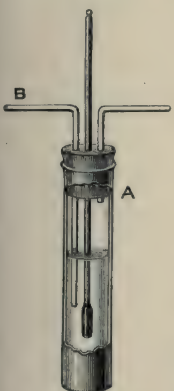


FIG. 88.—Principle of Regnault's hygrometer.

EXPT. 95.—**Dew point (by Regnault's hygrometer).** The principle of this hygrometer differs from that of the aluminium-cup hygrometer only in the method adopted for cooling the air. The lower end of a thin glass test-tube is fitted with a polished silver "thimble"; ether is poured into the tube, and its temperature is lowered by the evaporation caused by passing a stream of air bubbles through it until dew is deposited on the silver

thimble. Fig. 88 represents a simple device based upon this principle. The upper end of a brightly polished cylinder A (of brass or copper) is closed with a 3-holed cork. Through the cork pass (i) a thermometer, (ii) a glass tube B terminating below the surface of the ether contained in the cylinder, and (iii) a short glass tube for the escape of ether vapour. The ether may be cooled by breathing gently down the tube.

CHAPTER XII.

CONDUCTION. RADIATION. MECHANICAL EQUIVALENT OF HEAT.

Conduction of heat.—All metals are more or less good conductors of heat, but they differ very considerably in this property.

The **coefficient of conductivity** of any given metal is defined as **the quantity of heat which passes in one second through a 1 cm. cube of the metal between opposite faces which are maintained at a difference of temperature of 1° C.** When a temperature difference of θ°C. is maintained between opposite ends of a regular-shaped block of the metal of cross-section A sq. cm. and length L cm. the quantity Q of heat transmitted through the block will be proportional to (i) the coefficient of conductivity (k) of the metal, (ii) to the fall of temperature per unit length, *i.e.* to θ/L, (iii) to the cross section A, and (iv) to the time t. Hence

$$Q = \frac{k \times \theta A t}{L},$$

or

$$k = \frac{Q \times L}{t \times A \times \theta}.$$

EXPT. 96.—**Relative conductivity of metals.** The apparatus designed by Mr. E. Edser provides an efficient method of measuring relative conductivity. It consists of a copper vessel (A, Fig. 89) through which steam can be passed. Several rods of different metals of equal dimensions are soldered through holes in the bottom of the

vessel. Each rod is fitted with an index (B) of twisted wire, which slides freely down the rod. In the first instance each index is fixed to the top of its rod by a thin coating of paraffin wax, and, when the vessel is heated, the indexes slide downwards until they reach points which have the same temperature as that at

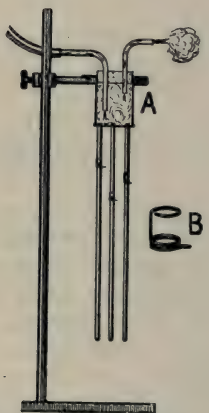


FIG. 89.—Relative conductivity of metals.

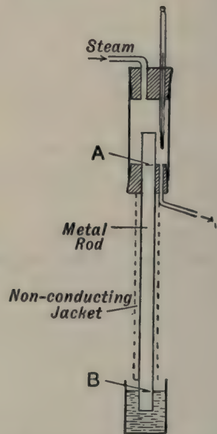


FIG. 90.—Absolute conductivity of a metal.

which the wax solidifies. The final distance of each index below the vessel should be measured. It can be proved theoretically that the relative conductivities are proportional to the *squares* of these distances.

EXPT. 97.—**The absolute conductivity of a metal.** A suitable form of apparatus is shown in Fig. 90. A cylindrical rod, about 20 cm. × 0·8 cm., is supported in the lower rubber bung of a steam heater, which can be made conveniently from a piece of wide glass tubing. In order to minimise loss of heat by radiation from the surface of the rod, it should be wrapped round with a

layer of cotton wool; and, outside this, it is an advantage to wind spirally a strip of sheet-rubber. Even with this protection the loss of heat affects the result seriously, and the numerical value obtained may be from 30% to 50% too small.

Measure the length (L) between the two transverse scratches A and B (Fig. 90), and also between B and the lower end of the rod. Measure accurately the average diameter of the rod, and calculate its cross-section (A). Insert the rod into the rubber bung so that the scratch A coincides with the inside surface of the bung. Put on the non-conducting jacket of cotton wool, and erect the apparatus. Support a dish of cold water below the rod so that the rod is immersed just to the level of the scratch B. Pass steam through the steam heater, and allow this to proceed for at least 10 minutes, by which time a steady temperature-gradient will have been established between the points A and B. Meanwhile weigh a calorimeter (i) when empty, and (ii) when containing about 100 c.c. of cold water. It is desirable that the temperature of the water should be about 2° C. cooler than that of the room. Protect the calorimeter with a layer of cotton wool.

Note the temperature (T_1) of the steam, also the temperature (t_1) of the water in the calorimeter, using for this purpose a thermometer reading to 0°·2 C. With a watch (having a seconds hand) on the bench, quickly substitute the calorimeter for the dish, note the time when the change is made, and adjust the calorimeter so that the mark B again coincides with the water surface. Keep the water gently stirred, and note the temperature at the end of every two minutes until about ten minutes have elapsed. Calculate the water-

equivalent (i) of the calorimeter, and (ii) of the length of the metal rod immersed; add these to the weight of water taken. Enter your observations thus:

Length of rod between the marks (L) = ... cm.
Average diameter of rod = ... "
Cross-section of rod (A) = ... sq. cm.
Temperature of room = ...° C.
Temperature of steam (T_1) = ...° C.
Initial temperature of cold water (t_1) = ...° C.
Final temperature of cold water (t_2) = ...° C.
Duration of experiment (t) = ... sec.
Total weight of water (W) = ... gm.
Quantity of heat transmitted (Q) = $W(t_2 - t_1)$ = ...
Average temp. difference between A and B,

$$(\theta) = T_1 - \frac{t_1 + t_2}{2} = \ldots\ldots$$

Calculate the value of k from the equation

$$k = (Q \times L)/(t \times A \times \theta).$$

EXPT. 98.—**Conductivity of a bad conductor.** Fig. 91 represents a simple method of measuring the conductivity

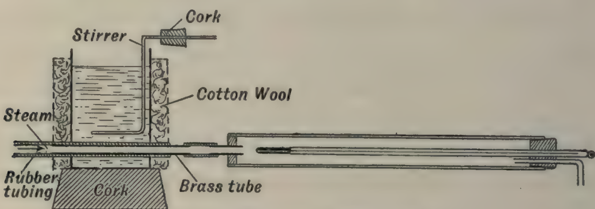


FIG. 91.—Conductivity of indiarubber.

of rubber: the same method could be used for glass. A piece of black rubber tubing, ¼ in. diameter, is slipped over a piece of *thin-walled* brass tubing, the diameters being such that the rubber fits tightly. This passes through two round holes bored in opposite sides of a

calorimeter (previously weighed); if the holes are carefully bored the calorimeter will be water-tight. When steam is passed through the brass tube heat will be transmitted through the rubber to the surrounding water; as the conductivity of brass is six hundred times greater than that of rubber, the presence of the brass tube does not affect the result appreciably. The temperature of the steam is observed by means of a thermometer supported horizontally in a long glass tube attached to the steam exit. The sequence of observations is as follows:

Weigh the calorimeter and stirrer, and calculate their water equivalent. Measure, with a micrometer gauge, the diameter of the brass tube (d_1) and of the rubber tubing (d_2) after it is fixed on the brass tube; the thickness of the rubber will be $(d_2 - d_1)/2$. Measure the internal diameter (L) of the calorimeter. The *average* cross-section of the rubber across which the heat is transmitted will be $L \times \pi(d_2 + d_1)/2$. Place the tube in position, through the holes in the calorimeter, and fit up the apparatus as shown in Fig. 91. Pour in a measured quantity (say, 100 c.c.) of water cooled to about 8° C. Connect the tube to the source of steam. Note the temperature (i) of the steam (T), and (ii) of the room (t). Note the time when the temperature of the water is $(t - 5)$° C. and when it is $(t + 5)$° C.

If the *total* weight of water is W grams, then

$$\text{calories of heat transmitted per sec.} = \frac{W \times 10}{\text{time (in secs.)}}.$$

Finally, calculate k from the equation

$$k = \frac{\text{calories per second} \times (d_2 - d_1)/2}{\{L \times \pi(d_2 + d_1)/2\} \times (T - t)}.$$

As rubber tubing does not consist entirely of pure rubber, the result obtained cannot be expected to agree with the recognised figure (0·00045) for pure rubber. But it is quite easy to obtain by this simple experiment a result approximating to 0·00030.

EXPT. 99.—**Radiating power of different surfaces.** Select two calorimeters of the same metal and size. Polish the surface of one, and paint that of the other with a *dead-black* paint.[1] Fit to each of these a cork bung, pierced with a single hole to carry a thermometer; also fasten to the bottom of each vessel three small feet of cork. Pour into each vessel the same volume of hot water, place them on the bench some distance apart and protected from draughts, and note every minute the readings of the thermometers. Continue the readings for at least thirty minutes. Plot the readings on squared paper, taking times as abscissae and temperatures as ordinates. Note also the temperature of the room.

Record the observations thus:

Temperature of room, 64° F.

Time.	Black vessel.	Bright vessel.
0	180° F.	—
0·5	—	182° F.
1·0	176°·5	—
1·5	—	179°·7
2·0	173°·2	—

Fig. 92 represents the type of cooling-curves which should be obtained.

[1] A good dead-black paint can be made readily by making a *dilute* solution of shellac in methylated spirit, and adding to it a sufficient quantity of *vegetable-black* (or lamp-black may be used). It is an advantage to add a little Venice turpentine, which serves to toughen the shellac.

If time allows, the student may advantageously repeat the experiment with a series of vessels protected by various

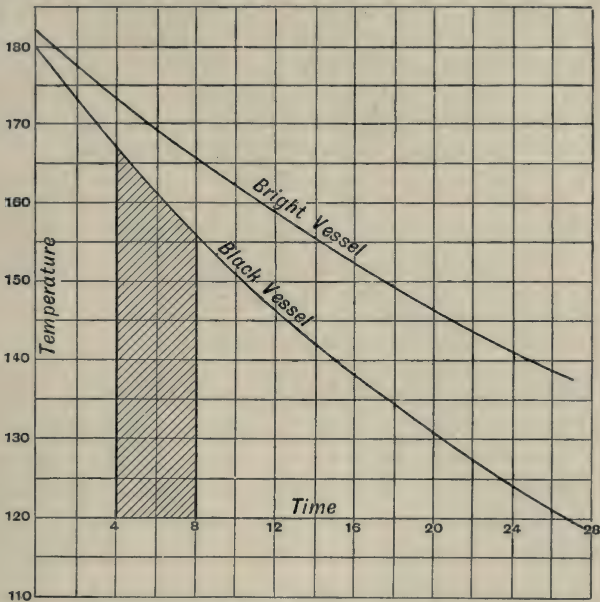


FIG. 92.—Cooling curves of polished and dull surfaces.

non-conducting jackets, *e.g.* cotton wool, sheet asbestos, asbestos wool, flannel, rubber, etc.

EXPT. 100.—**Newton's Law of Cooling.** With the aid of the cooling-curve obtained with the black vessel in the previous experiment find, for any selected period of time, *e.g.* four minutes, the value of the ratio

$$\frac{\text{fall of temperature}}{\text{mean temp. difference between vessel and the room}}.$$

Select for the same purpose other four-minute periods at various points of the curve, and find whether the value

of the ratio is constant. Examine the results for information as to whether Newton's law is true under all conditions, or whether it is true only when the mean temperature difference between the vessel and the room is small.

Fig. 92 suggests how the time period (4 min.–8 min.) may be investigated. Tabulate your figures thus:

(i) Time period.	(ii) Fall of temp.	(iii) Mean temp.	(iv) (iii) – temp. of room.	Ratio $\frac{\text{(ii)}}{\text{(iv)}}$.
(4 min.-8 min.)	167° – 156° = 11°	161°·5	97°·5	0·112

EXPT. 101.—**Specific heat by method of cooling.** Weigh an empty vessel, as used in Expt. 99, and nearly fill it with a measured volume of water. Weigh the vessel and its contents. Warm the vessel until its temperature is about 50° C. Support it on a non-conducting surface and screen it from air currents. Allow it to cool, and note the time required to cool (i) from 45° C. to 35° C., and (ii) from 35°C. to 25° C. Empty and dry the vessel. Pour into it the same volume of the liquid of unknown specific heat, and weigh the vessel and its contents. Raise the temperature to about 50° C., and observe the time-intervals required for it to cool through the same ranges of temperature. Calculate the specific heat of the second liquid from each range of cooling.

If w_1 be the weight of water taken, and e the water equivalent of the vessel, the heat lost in cooling from $T_2°$ to $T_1°$ is $(w_1+e)(T_2-T_1)$ calories. If w_2 be the weight of the other liquid, and s is specific heat, the heat lost is $(w_2s+e)(T_2-T_1)$ calories. If t_1 and t_2 be the time-

intervals required in the two cases, then, since *the quantity of heat lost is proportional to the time occupied,*

$$\frac{(w_1+e)(T_2-T_1)}{(w_2s+e)(T_2-T_1)}=\frac{t_1}{t_2}, \quad \text{or} \quad s=\frac{w_1t_2+e(t_2-t_1)}{w_2t_1}.$$

EXPT. 102.—**The mechanical equivalent of heat.** Obtain a cardboard tube, about 1 metre long and 5 cm. diameter, fitted with a cork at each end. Weigh out about 500 gm. of small lead shot contained in a dish or beaker, and observe the temperature (t_1) of the shot, using a thermometer reading to 0°·2 C. Remove one of the corks; and, holding the tube almost horizontal, transfer the shot to the tube. Hold the tube vertically, and measure by means of a metre scale the distance from the top of the lead shot to the top of the tube. Subtract from this the length of the tube occupied by the upper cork when firmly inserted. This gives the distance h through which the shot are raised when the tube is inverted. Insert the upper cork, and, with a hand at each end of the tube (so as to prevent the corks from being dislodged), rapidly invert it to a vertical position. Repeat this movement at least 50 times, counting the number n of times the inverting has been repeated. Transfer the shot back to the dish, at once insert the thermometer into the shot, and note the final temperature (t_2).

If w gm. = weight of shot,
h cm. = vertical distance within the tube,
n = number of times the tube is inverted, then
mechanical work done $= n \times wh$ gm.-cm. units.

If s = specific heat of lead,
$(t_2-t_1)°$ C. = rise in temperature, then
heat developed $= ws(t_2-t_1)$ calories.

Hence, **mechanical work equivalent to one calorie** $\Big\} = \dfrac{n \times wh}{ws(t_2-t_1)}$ gm.-cm.

The work done by an electrical current flowing along a conductor is measured in terms of a unit called a **joule**, which is **the work done by a current of 1 ampere flowing for 1 second between two points of a conductor which differ in potential by 1 volt.** When, therefore, a current of C amp. flows for t sec. between two points which differ in potential by E volts, the work done is E Ct joules. Since 1 joule $= 10^7$ ergs, the work done may be expressed as $(ECt \times 10^7)$ ergs.

In a simple electrical circuit, in which no mechanical work is done, this work reappears in the form of heat, in an equivalent quantity. This heat can be measured by having the conductor immersed in a known weight of water. If the quantity of heat measured is H calories, then

$$\left.\begin{array}{c}\textbf{the mechanical equivalent}\\ \textbf{of 1 calorie}\end{array}\right\} = \frac{ECt \times 10^7}{H} \text{ ergs.}$$

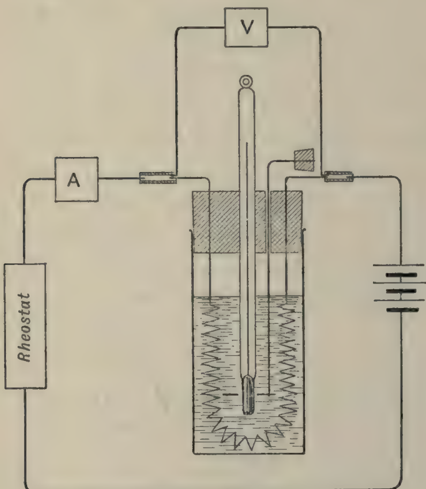


Fig. 93.—The mechanical equivalent of heat (by electrical method).

In Fig. 93 the conductor consists of a spiral of manganin wire (S.W.G. No. 22) having a resistance of about 5 ohms. This is mounted within a copper calorimeter (capacity, about 250 c.c.). The cork is fitted with a thermometer, reading to 0°·2 C., and a stirrer. V is a voltmeter (0-10 volts), and A is an ammeter (0-3 amp.). A uniform current is maintained by means of a *rheostat* included in the circuit. The most suitable source of current is three or four accumulators.

EXPT. 103.[1]—**The mechanical equivalent of heat (electrical method).** Weigh the empty calorimeter, calculate its water equivalent, add ice-cold water sufficient to cover the spiral, and again weigh. Insert the spiral, protect the calorimeter with cotton wool, and complete the circuit as shown in Fig. 93; adjust the rheostat until the readings on the ammeter and voltmeter are of appropriate magnitude. Break the circuit.

Decide upon a suitable range of temperature; thus, if the temperature of the room is $t°$ C., it will be appropriate to determine exactly the length of time required to warm the water from $(t-4)°$ C. up to $(t+4)°$ C. Now close the circuit, and stir the water regularly; watch the thermometer, and note the time when the temperature is $(t-4)°$ C. Note the readings of the ammeter and of the voltmeter. Alter the rheostat if necessary, so as to keep the current constant; and, if the voltmeter reading varies, note the reading every minute. Meanwhile, keep the water stirred, and note the exact time when the temperature $(t+4)°$ C. is reached. You have now all the data necessary for calculating the mechanical equivalent by means of the equation given above.

[1] This experiment should be carried out only by students who have taken previously an elementary course in Electricity.

LIGHT.

CHAPTER XIII.

PHOTOMETRY. REFLECTION AND REFRACTION AT PLANE SURFACES.

PHOTOMETRY.

EXPT. 104.—**Rumford's shadow photometer.** Fit up the apparatus shown in Fig. 94, where C is a *standard*[1] candle, and L is an electric lamp, or any other source of light, of which the *candle-power* is to be measured.

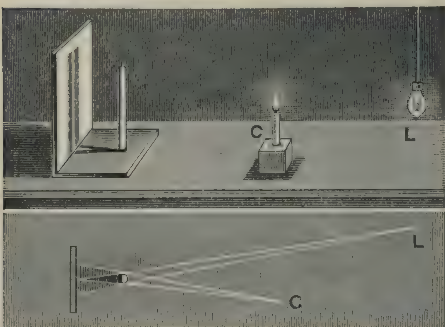


FIG. 94.—Rumford's shadow photometer.

Adjust the positions of C and L so that the two shadows just touch, but do not overlap, and so that the two shadows are equal in *depth* of tint. Measure the distances of L and C from the shadows, and use the equation $L_1/L_2 = d_1^2/d_2^2$ in order to find the candle-power of the lamp. In the same way find the candle-power of (i) an ordinary wax candle, (ii) a paraffin lamp, (iii) a

[1] The 'standard candle' is of spermaceti, weighing six to the pound, and burning at the rate of 120 grains per hour.

fish-tail gas burner. If the two shadows differ considerably in *colour* it is not easy to adjust them to equal depth of tint. In such circumstances the adjustment is much easier if the shadows are viewed through a piece of neutral-tinted glass.

The student should remember that (in Fig. 94) the shadow on the left is illuminated by C only; and the intensity of illumination is proportional to L_1/d_1^2 where L_1 is the candle-power of the candle, and d_1 is its distance from the screen. The shadow on the right is illuminated by L_2 only; and the intensity of illumination is proportional to L_2/d_2^2. When the two shadows are equally bright, then $L_1/d_1^2 = L_2/d_2^2$. From this is derived the equation used in the experiment.

EXPT. 105.—**Bunsen's grease spot photometer.** This experiment should be carried out in a perfectly dark room.

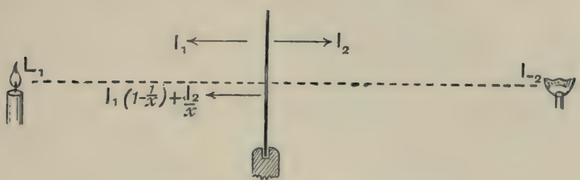


FIG. 95.—Bunsen's grease spot photometer.

The apparatus consists of a screen of opaque white paper having a small part of its surface greased so as to render it slightly transparent. The two sources of light to be compared are placed on opposite sides of the screen, as shown at L_1 and L_2 (Fig. 95). Adjust the relative distances, d_1 and d_2 of the sources from the screen until the greased and ungreased parts of the screen appear to be equally bright. Having taken these measurements when viewing one side of the screen, repeat the measurements when viewing the other side of the screen; and,

if there is any difference in the measurements, take the *average* distances as being correct.

Calculate the unknown candle-power by means of the equation $L_1/d_1^2 = L_2/d_2^2$.

Let I_1 and I_2 be the intensities of illumination of the opposite side of the opaque paper. If $1/x$ be the fraction of the incident light which is *transmitted through* the grease spot, then $\left(1-\frac{1}{x}\right)$ is the fraction *reflected from* the grease spot. Hence, when the screen is viewed from the left-hand side, the apparent illumination of the grease spot will be $I_1\left(1-\frac{1}{x}\right)+I_2/x$. If adjusted so that the illumination of the grease spot is the same as that of the surrounding paper,

then $$I_1 = I_1\left(1-\frac{1}{x}\right)+I_2/x,$$

or $$I_1/x = I_2/x,$$

or $$I_1 = I_2,$$

or $$L_1/d_1^2 = L_2/d_2^2.$$

Reflection from Plane Surfaces.

Expt. 106.—**The angles of incidence and of reflection are equal.** Fasten a sheet of white paper on a drawing board, and support in a vertical position on the paper a strip (about 2 inches by 1 inch) of good plane mirror AB Fig. 96). (The mirror may be supported by fixing

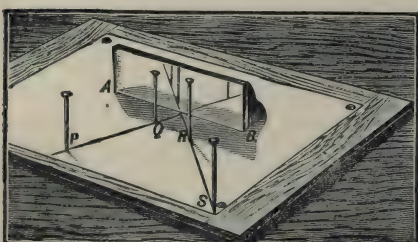


Fig. 96.—Method of demonstrating equality of angles of incidence and reflection.

a small wooden cube to the back of the mirror with wax.)

Fix two pins P and Q, vertically in the board and approximately in the positions shown. View the images of these pins by looking in the direction SR, and move the eye until the image of Q overlaps that of P. Keep the eye in this position, and insert two other pins at points such as R and S, and so that these pins together with the images of P and Q all appear to be in one straight line. Draw a pencil line at the back of the mirror.

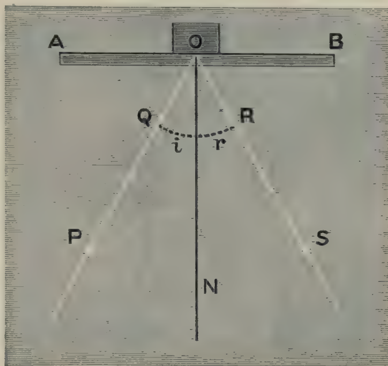


FIG. 97.—Construction to illustrate reflection of light at a plane surface.

Remove the mirror; draw the lines PQ and SR and produce them until they meet (Fig. 97). They should meet at a point O approximately on the line of the silvered surface. Draw a line ON normal to the mirror at O. Measure the angle of incidence PON, and the angle of reflection SON, and compare them. Repeat the experiment two or three times, with different angle of incidence in each case.

EXPT. 107.—**Relative positions of image and object.** Support a strip of plane mirror AB (Fig. 98) in a vertical position on a sheet of paper, and fix a pin vertically at O. View the image of the pin in the direction PQ, and insert pins at P and Q. Similarly, view the image in the direction RS, and insert pins at R and S. The image is situated somewhere along the

line PQ produced, and also somewhere along the line SR produced. Hence it can only be at the point where these lines intersect.

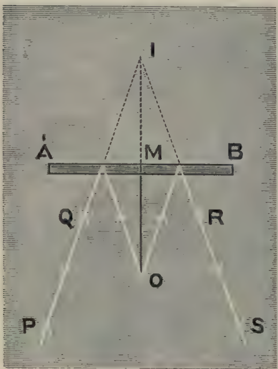


FIG. 98.—Expt. 107.

Remove the mirror, and produce the lines PQ and SR to meet at I. Draw normals to the mirror from I and O. These normals should be in the same straight line, and equal in length.

EXPT. 108.—**Parallax method of locating image.** Use the same mirror as before. If it be 1 inch high, use pins about 2 inches long. Insert a pin at O (Fig. 98), and view its image just to one side of the normal. Fix a pin O′ in the paper behind the mirror so that the upper part of this pin may appear to be a continuation of the image of the pin at O. Move the eye slightly to the right and then to the left; if the upper part of O′ appears to move relatively to the image of O in the *same* direction as the eye, then O′ is *too far* from the mirror. Adjust the position of O′ until, from whatever direction it is viewed, it always appears to be continuous with the image of O. *The pin* O′ *then occupies the position of the image of* O.

REFRACTION AT PLANE SURFACES.

EXPT. 109.—**The refractive index of glass (pin method).** Spread a sheet of white paper upon a drawing board, and draw a straight line on the paper. Lay a thick glass slab (about $10 \times 8 \times 1{\cdot}5$ cm.; *rectangular edges*)

on the paper with its edge AB (Fig. 99) upon this line. Close to the edges CD and AB insert pins P and Q, so that the line PQ is oblique to AB. Look at P *through the block*, and move the eye until Q covers the image of P. Insert a third pin, R, in such a position that it appears to be in line with P and Q.

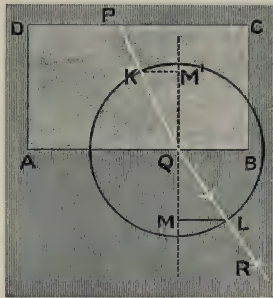


FIG. 99.—Refraction by glass.

Remove the block; draw the normal MM′; with Q as centre, describe a circle, 4-5 cm. radius. Drop perpendiculars LM and KM′, and measure their lengths. The ratio LM/KM′ is the *refractive index* (μ) of glass.

Replace the block, alter the position of the pin P, and make another determination of μ.

EXPT. 110.—**Refractive index of glass, by locating the image due to refraction at a plane surface.** Lay a slab of glass ABCD (Fig. 100) on a sheet of paper, and fix a pin vertically at O in contact with one edge of the glass. View the pin through the glass from a position along the normal ON and beyond the opposite edge. That portion of the pin which is seen through the glass will appear to be at some point I. Determine the position of this point by holding another pin vertically with its point touching the glass and moving the point to-and-fro along ON until a position is found where the movable

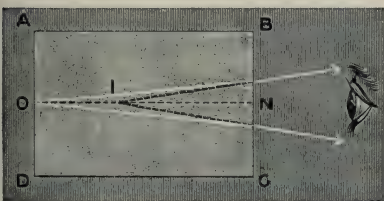


FIG. 100.—Expt. 110.

pin appears to remain continuous with the image of O when the eye is moved slightly to right and left.

Measure the distance IN and ON, and calculate the refractive index of glass. The determining of the position of I is facilitated when a narrow strip of gummed paper is fastened to the glass along the line ON.

EXPT. 111.—**Refractive index of water, by measurement of apparent depth**. Fill a deep glass cylinder with water, and drop to the bottom of the cylinder a small opaque object (*e.g.* a short piece of thick copper wire). Clamp a narrow glass tube, drawn out to a jet, vertically above the surface of the water. Connect the tube to the gas supply, fix it so that the jet is horizontal, and light the gas at the jet, adjusting the supply so that a *small* yellow flame is obtained.

View the arrangement vertically downwards and observe whether there is any parallax between the immersed object and the image of the flame reflected from the surface of the water. Adjust the distance of the jet above the water until there is no parallax.

In this position the apparent depth of immersion of the object must coincide with the position of the flame's image, and the distance of the latter below the surface is equal necessarily to the vertical height of the jet above the surface. Take the necessary measurements, and calculate the refractive index.

EXPT. 112.—**Refraction through a prism**. Place a prism ABC (Fig. 101) upon a sheet of paper, and fix two pins in positions corresponding to D and E. View the two pins through the prism in the direction D′E′. Slowly rotate the prism round the point A, and in either direction: notice that the line D′E′ of the image varies, and

that there is *one position* of the prism in which D′E′ approaches most nearly to the direction GF of the incident ray. Mark the emergent ray by means of pins D′ and E′, and trace the outline of the prism on the paper. Remove the prism and pins.

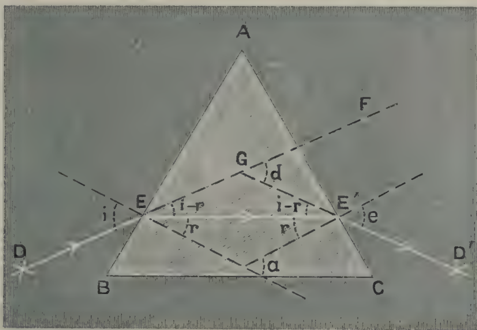


FIG. 101.—Refraction of a ray of light through a prism.

Draw the incident ray DE and the emergent ray E′D′. Join EE′. The path of the ray is represented by DEE′D′. Note whether EE′ is parallel to the base BC. Produce DE to any point F, and D′E′ to G. Measure the angle of deviation (**d**). Draw normals at the points E and E′, and measure the angles of incidence (**i**) and of emergence (**e**). The angle A of the prism is equal, necessarily, to the angle (**a**) between the normals at E and E′. Measure the angle (**a**) of the prism, and calculate the refractive index of glass by means of the equation

$$\mu = \sin\frac{\mathbf{d}+\mathbf{a}}{2} \div \sin\frac{\mathbf{a}}{2}.$$

CHAPTER XIV.

MIRRORS AND LENSES.

Spherical mirrors.—When a narrow beam of rays parallel to the principal axis of a spherical mirror is reflected from the surface of the mirror, the rays converge to, or diverge from, a point on the principal axis. This point is termed the **principal focus**. The distance of the principal focus from the pole of the mirror is termed the **focal length** of the mirror.

Since the radius of curvature of a spherical mirror is equal to *twice* the focal length, the measurement of the former serves as a method for determining the latter.

Alternative methods are based upon the *general equation* for mirrors,

$$\frac{1}{\mathbf{f}}=\frac{1}{\mathbf{v}}+\frac{1}{\mathbf{u}},$$

where **u** and **v** are the distances of the object and image respectively from the mirror, and **f** is the focal length. In using this equation it is necessary to apply always the following rule as to the signs *plus* and *minus*: All distances measured from the mirror *towards* the source of light are **positive**, but distances measured from the mirror and *away from* the source of light are **negative**.

Fig. 102 suggests some simple appliances which are appropriate for measurements of focal lengths. Fig. 102 (*a*) is an incandescent gas burner with an iron chimney in which a circular hole is bored. Fig. 102 (*b*) is a piece of cardboard with circular hole and cross-wires; this is supported vertically in front of the hole in the gas chimney.

When the *parallax method* of locating the image is used, narrow cardboard strips with a pencil line on each, Fig. 102 (*c*) and (*d*), are suitable, the former serving as the object, and the latter as a means of locating the image. Of course, an **optical bench** is very convenient for certain methods, but it is not essential. When a special support for mirrors

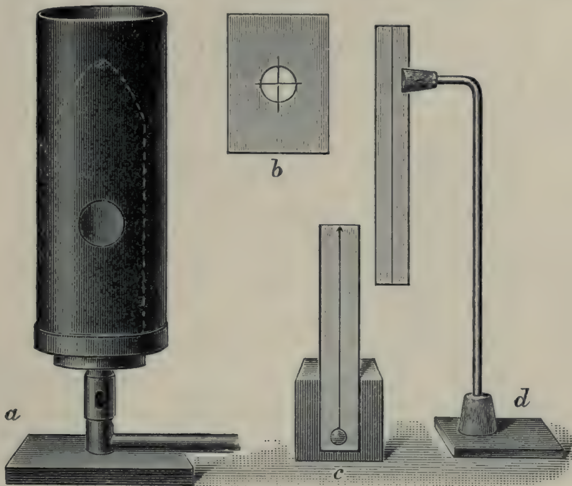


FIG. 102.—Apparatus for experiments with mirrors and lenses.

(or lenses) is not provided, these may be fixed vertically on a rectangular wood block by means of soft wax—or *Harbutt's plasticene* is very efficient for the purpose.

In all forms of optical bench, distances from the mirror (or lens) to the cross-wires and to the screen are observed by means of a scale attached to the baseboard, and the scale-readings of the stands carrying the various parts are observed. As the distance apart of the stands is not necessarily the same as that between the items supported by the stands, the following procedure should be adopted: measure the length of a long steel knitting needle, hold it horizontally with one end just touching the cross-wires,

and move the mirror (or lens) until the reflecting (or refracting) surface just touches the other end of the needle. Read the difference between the scale-readings of the two stands; the true distance apart is equal to the length of the needle, and the difference between this and the apparent distance as read off on the scale must be applied always as a *correction* (either + or −) to the distances measured on the scale. The correction must also be found for the observed distance between the mirror (or lens) and the screen.

EXPT. 113.—**Focal length of a concave mirror.** (i) Allow a beam of parallel rays to fall upon the mirror, and adjust the position of a piece of white cardboard so that a well-defined image is formed on its surface. The distance of the cardboard from the pole of the mirror is the focal length of the mirror. A beam of sunlight is convenient for this experiment; or a distant chimney, or window frame, may be used.

(ii) Using the illuminated cross-wires (Fig. 102 *b*), adjust the position of the mirror so that a well-defined image of the cross-wires is seen upon the cardboard to which the cross-wires are attached, and near to the round hole.

Evidently the rays of light are reflected back along their own path, and the cross-wires must be situated at the centre of curvature of the mirror. Measure this distance; one half of this distance is the focal length.

Providing that the room is partially darkened, the surfaces of a bi-concave lens may be used as mirrors, both in this experiment and in the following.

(iii) Using the same source of light as in the preceding experiment, adjust the mirror so that its principal axis makes a small angle with the axis of the incident beam. Hold a piece of white cardboard vertically and normal to the reflected beam, and adjust its position so that a well-defined image of the cross-wires is seen on its surface.

Measure the distance **u** of the cross-wires from the mirror, and the distance **v** of the image from the mirror; calculate the focal length by means of the formula

$$\frac{1}{\mathbf{f}}=\frac{1}{\mathbf{v}}+\frac{1}{\mathbf{u}}.$$

Make at least three independent experiments, using in each case a different value for **u**.

(iv) Find the focal length of a concave mirror by the *parallax method*, using the cardboard strips and pencil

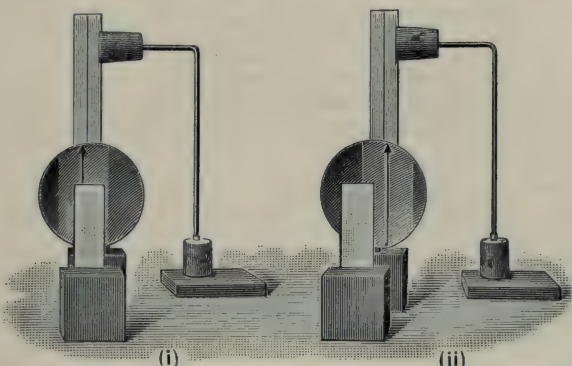


FIG. 103.—Parallax method, for a concave mirror: when the eye is moved (i) towards the left, (ii) towards the right.

lines (Fig. 102 (*c*) and (*d*)), or long needles mounted vertically on corks. Place the object near enough to the mirror to obtain an *upright enlarged* image when viewed with the eye along the axis of the mirror, and adjust the height of the object so that the image extends to the upper edge of the mirror. Place the other pencil line *behind* the mirror, and adjust its position until, on moving the eye to right and to left, this pencil line appears always to be continuous with that observed in the image. Fig. 103 (i) and (ii) represent observations which show that the

image is *more distant* than the finder. Measure the distances **u** and **v**. Make at least two other observations, with the object at greater distances from the mirror. Calculate the focal length from each set of observations.

Tabulate the results thus:

u.	v.	f (calculated).
+15	−29·1	30·95
+53·55	+73·0	30·89
+90·8	+46·7	30·84

On squared paper draw to scale one or more of these observations. Taking the *average* value for the focal length found in the several experiments, and assuming the

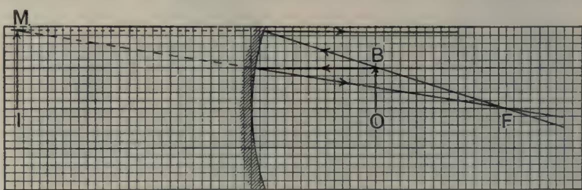


Fig. 104.—Graphical method of locating the image formed by a concave mirror.

magnitude of **u** as measured, find geometrically the magnitude of **v**. Fig. 104 is the diagram corresponding to the first of the above measurements, and OB represents an imaginary object 15 cm. distant from the mirror. From the point B two rays of light are drawn, viz. one ray parallel to the axis and reflected back through the focus F, and the other passing along FB and reflected back parallel to the axis; the point of intersection of the two reflected rays is the position of the image M of the point B.

Expt. 114.—**Focal length of a convex mirror.** (i) Let O (Fig. 105) represent the position of the illuminated cross-

wires, and let L be a bi-convex lens. The rays of light passing through L will converge to a point I, where a well-defined image will be formed upon a cardboard screen. Adjust the position of the screen until the best possible definition is obtained. Support

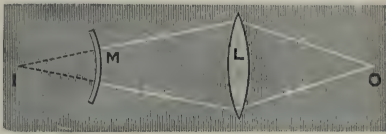


FIG. 105.—Experimental method of determining focal length of a convex mirror.

the mirror M in a vertical position between L and I, and adjust its position until a well-defined image of the cross-wires is formed upon the cardboard at O.

Since the rays are now reflected back along their own path they must fall normally upon the surface of M, and the distance IM between the screen at I and the mirror M must be the radius of curvature of the mirror. One half of this distance is the focal length.

(ii) Find the focal length of a convex mirror by the *parallax method*. Using the cardboard and pencil-line appliance, let the object line be sufficiently long, or raised enough, so that the image extends to the upper edge of the mirror. As the image is always virtual, the finder line must be placed *behind* the mirror; having located the image, take the measurements of **u** and **v**, and calculate the value of **f**, remembering that in this case **v** is negative. Repeat the experiment at least twice.

Draw at least one diagram to scale and, by this means, verify the result obtained experimentally for the position of the image.

Lenses.

Methods of finding focal length.—The simpler available methods of finding the focal lengths of lenses may be tabulated as follows:

(A) **Converging Lenses.**—(*a*) *Parallel beam of light methods*: (i) where the parallel beam is obtained by direct rays from the sun, or from a very distant object; (ii) where a parallel beam is reflected from a plane mirror back through the lens.

(*b*) *Parallax method*, where the measurements obtained are applied to the general equation $1/\mathbf{f} = 1/\mathbf{v} - 1/\mathbf{u}$. This method is applicable when the image is either real or virtual.

(*c*) *Optical bench methods*, in which a real image of a luminous object is always obtained, and the measurements applied to the above general equation, or to one deduced from it.

(B) **Diverging Lenses.**—(*a*) *Parallax method.*

(*b*) *Optical bench method*, in which the lens is combined with a converging lens of shorter focal length, and the focal length of the combination measured.

When calculating the focal length from measurements of $\mathbf{u}$ and $\mathbf{v}$, the distances respectively of the object from the lens and of the image from the lens, it is necessary to apply the **Rule of Signs**, which states that **Distances measured from the lens towards the source of light are positive, and distances measured from the lens and away from the source of light are negative.** A consideration of the effect of each type of lens on a parallel beam of light shows that the focal length of a *converging* lens is always *negative*, and that of a *diverging* lens is always *positive*.

The appliances required for the experimental determination of focal lengths are the same as those previously used for mirrors; in the parallax method it may be advantageous in some cases to use as a finder a long needle fixed vertically in a cork instead of the pencil line on cardboard.

EXPT. 115.—**Converging lens: parallel beam method.** (i) Support the lens in a vertical position and adjust its distance from a cardboard screen so as to form on the screen a real inverted image of some distant object (*e.g.* the window bars, or a distant chimney). The rays from the distant object are practically parallel, and the image is formed at the principal focus of the lens. Measure the distance of the screen from the lens.

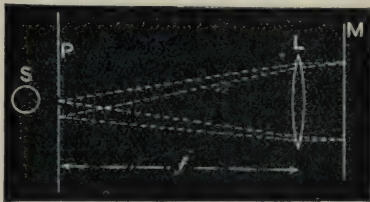


FIG. 106.—Determination of the focal length of a convex lens.

(ii) In front of a bright light clamp vertically the cardboard screen with cross-wires. Support the lens so that the centre of the hole is on the principal axis of the lens. Place a plane mirror M (Fig. 106) close behind the lens, and adjust the distance of the lens L from the screen until an image of the cross-wires is formed on the screen P and close beside the circular hole.

Evidently each ray of light is reflected back along its own path; and this can only be the case if the rays fall normally on the surface of M. Hence, the rays after passing for the first time through the lens are parallel to the principal axis, and they must therefore have originated from the principal focus. Therefore the distance *f* is the focal length of the lens.

EXPT. 116.—**Converging lens: parallax method.** Place the lens 10 cm. in front of the vertical pencil line (Fig. 102 *c*), and view the pencil line through the lens. Observe (i) whether the image is erect or inverted; (ii) whether it is magnified or diminished; (iii) by moving the eye to right and left, observing how the image moves relatively to the centre of the lens, note whether

the image is in front of the lens or behind it; (iv) by raising the eye until part of the object is visible above the top of the lens, and repeating the movement to right and left, observe whether the image is in front of the object or behind it; (v) by means of the finder locate the actual position of the image, and measure its distance, **v**, from the lens.

Tabulate your observations thus:

Distance of object from lens (**u**).	Is image erect or inverted?	Is it magnified or diminished?	Is it in front of, or behind, lens?	Is it in front of, or behind, object?	Distance of image from lens (**v**).
+10 cm.	erect	diminished	behind	in front of	+7·6 cm.

Repeat the observations when **u** is increased to 15, 20, 25 ... cm., and enter the information obtained in the above table. At one distance it will be observed that the image is practically invisible, but at a slightly increased distance the image will be real and inverted, and situated in front of the lens near to the observer's eye. The position of the image can be located readily by the *finder*, and care must be taken to attach to **v** its proper sign.

From each of the above observations calculate the focal length of the lens by means of the equation $1/\mathbf{f}=1/\mathbf{v}-1/\mathbf{u}$.

Draw, on squared paper, diagrams to scale of two of the observations, one in which the image is virtual and the other in which the image is real.

EXPT. 117.—**Converging lens: optical bench methods.** (i) Erect the cardboard screen and cross-wires, and adjust the lens so as to obtain a well-defined image of the cross-wires on a distant cardboard screen. Measure **u** and **v**, and calculate the focal length. Make at least three

independent sets of observations, the distance **u** being different in each case.

(ii) Adjust the cross-wires, lens and screen so as to obtain a well-defined image, as in the previous experiment. Let P_1 (Fig. 107) represent the position of the lens. Since O and I are *conjugate foci*, and **u** and **v** are mutually interchangeable, another position P_2 of the lens can be found where a well-defined image is obtained, although the distance between the cross-wires and screen remains unaltered. The focal length of the lens can be expressed in terms of D, the distance between the cross-wires and the image, and A, the distance between the two positions of the lens.

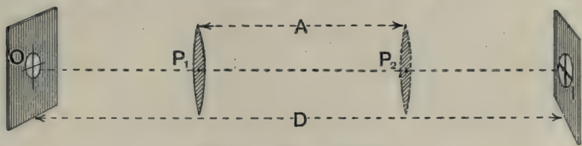


FIG. 107.—Expt. 117 (ii).

Thus, from the general equation $1/f = 1/v - 1/u$, when the lens is in the position P_1,

$$\frac{1}{f} = \frac{1}{-IP_1} - \frac{1}{OP_1} = \frac{OP_1 + IP_1}{-(OP_1 \, . \, IP_1)}.$$

But $$OP_1 = \frac{D-A}{2}$$

and $$IP_1 = D - OP_1 = \frac{D+A}{2}\text{; hence}$$

$$\frac{1}{f} = \frac{D}{-\left(\frac{D-A}{2} \cdot \frac{D+A}{2}\right)} = \frac{-4D}{(D^2 - A^2)},$$

or $$f = \frac{A^2 - D^2}{4D}.$$

Find by this method the focal length of the converging lens, using at least two different values of D.

EXPT. 118.—**Diverging lens: parallax method.** Make with a diverging lens a series of observations exactly similar to those described for a converging lens in Expt. 116; tabulate the observations in the same manner, and calculate the focal length of the lens from the general equation $1/f = 1/v - 1/u$. It may be found more convenient to use, as a finder, a thin needle supported vertically in a cork instead of the pencil line on cardboard as described on p. 153.

Draw on squared paper a diagram representing one of the sets of observations obtained.

EXPT. 119.—**Diverging lens: optical bench method.** Select a convex lens of known focal length, and of such converging power that, when fastened to the concave lens, the combination is still converging. If necessary, determine the focal length (f_1) of the convex lens, preferably by the method of Expt. 117 (ii). Fasten the two lenses together and determine the focal length (F) of the combination.

The focal length (f_2) of the concave lens can then be calculated by means of the equation $1/F = 1/f_1 + 1/f_2$. When substituting the values of f_1 and F it must be remembered that the focal length of a converging lens is always negative.

EXPT. 120.—**Model of an astronomical telescope.** Select two converging lenses, one having a long focal length to serve as the object-glass, and the other a much shorter focal length to serve as an eye-piece; erect the lenses vertically on corks. Fasten a sheet of large print, *e.g.* a poster, vertically at a distant part of the room, and locate by means of a *finder* the position of the real image of the print when viewed through the object-glass. Place

the eye-piece lens in line with the field lens, and adjust its position so that a well-defined image of the finder is observed when the eye is placed close to the lens. Remove the finder, and note whether a well-defined inverted image of the print can be seen.

EXPT. 121.—**Model of a microscope.** Select two converging lenses of short focal length: two-inch focal length is suitable. Erect the lenses vertically on corks. Clamp a millimetre scale in a vertical position, and place in front of it one of the lenses, at a distance slightly greater than its focal length. Locate the position of the image by means of a finder. Place the second lens in line with the first, and adjust its position so that a well-defined image of the finder is observed when the eye is close to the lens. Remove the finder, and note whether a well-defined magnified image of the scale can be seen.

CHAPTER XV.

SOUND.

EXPT. 122.—**The pitch of a tuning-fork (method 1).** Fig. 108 represents the essential details of a simple arrangement for the dropping-plate method of finding the pitch of a tuning-fork. The device consists of a sheet (about *7* in. × 3 in.) of thin glass, to the upper edge of which is cemented or glued a strip of wood somewhat longer than the width of the glass. Attach to one prong of the fork a narrow pointed style of thin sheet brass: the style can be prepared by beating with a hammer a small piece of brass sheet, until it becomes quite springy, and cutting off with scissors a narrow strip, about 0·5 in. long, tapering to a point: bend the style into a slight curve, warm the prong of the fork, and attach

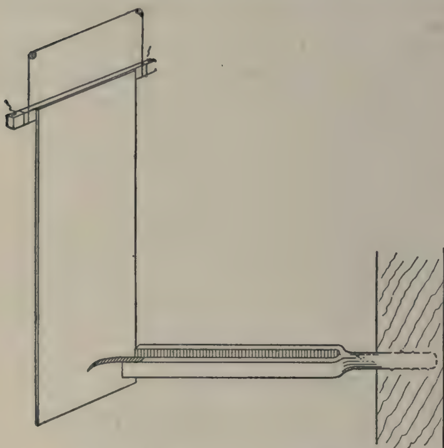


FIG. 108.—Dropping-plate apparatus.

to it the style by means of wax or bicycle-cement. Attach a loop of thread to the wood strip of the dropping plate, and hold the plate horizontally over burning camphor (or turpentine) until the surface is covered with a thin layer of soot.

Clamp the fork *rigidly* in a horizontal or sloping position, and suspend the plate so that a part near its lower edge touches the point of the style. Rock the plate slightly sideways so that the style makes a fine horizontal scratch on the blackened surface: this will be the starting point O (Fig. 109) for subsequent measurements. Draw a well-resined violin bow across both prongs of the fork and near their ends. As soon as the fork is sounding vigorously its fundamental note, burn the supporting thread. Examine the smoked surface for the waved line traced out by the style. Support the plate over a sheet of white paper, mark with a needle point the first and the last well-defined wave-crests (*a* and *b*, Fig. 109), and count the number (n) of crests *on one side of the waved line* between the points *a* and *b*.

Fig. 109.—Trace on smoked glass of tuning fork vibrations.

It follows that n vibrations of the fork were described in the time occupied by the plate in falling between the points *a* and *b*. Measure, with dividers and a millimetre scale, the distances O*a* and O*b*; denote these by s_1 and s_2. If t_1 and t_2 represent the time occupied in falling through the

distances Oa and Ob, then $(t_2 - t_1)$ is the time occupied in falling from a to b, and

$$(t_2 - t_1) = \sqrt{\frac{2}{g}}\,(\sqrt{s_2} - \sqrt{s_1}).$$

From this result calculate the number of vibrations which will be described in one second.

EXAMPLE.—Fig. 109 is a reproduction of a wave-trace obtained from a c' tuning-fork, and the following measurements were obtained:

$$Oa = 0{\cdot}29 \text{ cm.}$$
$$Ob = 18{\cdot}83 \text{ cm.}$$

Number of vibrations between a and b, 46.

Hence $\quad t_2 - t_1 = 3{\cdot}80/22{\cdot}15$ sec.,

and $\quad N = (46 \times 22{\cdot}15)/3{\cdot}80 = 268{\cdot}0$ vibrations per sec.

(The old Philharmonic Standard gives $c' = 271{\cdot}4$ vibrations per sec.)

The laws of vibrating strings.—The number of complete vibrations described in one second by a stretched string is expressed by the equation

$$n = \frac{1}{2l}\sqrt{\frac{\mathbf{F}}{m}},$$

where n is the number of vibrations per second, l the length of the string, **F** the stretching force, and m the mass per unit length of the string. When metric units are used, the length, force and mass must be expressed in centimetres, dynes and grams respectively.

The fact that the pitch of the note given by a stretched wire, *when sounding its fundamental note*, is

(i) inversely proportional to its length,
(ii) directly proportional to the square root of the stretching force, and
(iii) inversely proportional to the square root of the mass per unit length,

may be demonstrated by means of a sonometer (Fig. 110).

The sonometer should be provided with two stretched wires, one permanently attached and stretched by means of a piano-pin and key, the other removable and stretched by

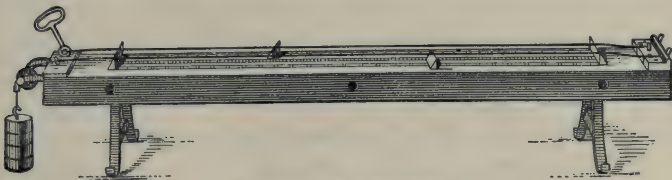


FIG. 110.—A sonometer.

means of heavy weights attached to its free end. It is advantageous to support the sonometer vertically on a wall, instead of horizontally as shown in Fig. 110.

EXPT. 123.—**Length of the wire.** (i) Tune the two wires until they are in unison.[1] Shorten by means of a movable bridge one of the wires A until it gives the octave above its original note as compared with the unaltered wire B. Its rate of vibration is twice its previous rate. Measure its length and note whether this is equal to one-half its previous length.

(ii) By means of another movable bridge tune the previously unaltered wire B until in unison with the shortened wire A. Move the bridge under A until its shorter position gives a note one octave above B, and therefore two octaves above its fundamental note. Its rate of vibration is now four times as great as its initial rate. Measure its length, and note whether this is equal to one-fourth of its initial length.

[1] The absence of *beats* is the most accurate means of detecting the exactness of unison. The number of beats per second is equal to the *difference* in the number of vibrations per second of the two sources of sound, when both are vibrating simultaneously; hence, unison is being approached when an adjustment reduces the number of beats per second. The beats are detected readily by the method suggested in Expt. 126.

(iii) If two tuning-forks of known rate of vibration are available, measure the lengths of one of the wires required to give notes in unison with the two forks respectively, keeping the tension the same. Note whether the ratio of the lengths is equal to the inverse ratio of the rates of the two forks.

EXPT. 124.—**The stretching force.** Stretch a thin wire on the sonometer with a weight of one kilogram, and tune the other wire to unison. Increase the stretching force to four kilograms, and find by comparison with the other wire whether the note now given is one octave above the previous note.

Try to verify the law when the stretching forces are weights of two and of three kilograms.

EXPT. 125.—**Diameter and material of the wire.** Select two wires (A and B) of different material, *e.g.* brass and steel, or two wires of the same material but of different diameter. Stretch one of them (A) with a known weight, and determine the length l_1 of the fixed wire C which is in unison with it. Make file marks on the wire A where it touches the bridges, unhang the weight, and cut the wire at the file marks by means of wire cutters. Weigh this length of wire, and determine the mass m_1 of unit length. Stretch the wire B with the same weight as before, and determine the length l_2 of the wire C which is in unison with it. Proceed, as before, to find the mass m_2 of unit length of wire B.

If n_1 and n_2 are the frequency of vibration of the wires A and B, then $n_1/n_2 = l_2/l_1$. Also, by the above equation, $n_1/n_2 = \sqrt{m_2/m_1}$. Calculate the values of the ratios l_2/l_1 and $\sqrt{m_2/m_1}$, and observe whether they are equal.

EXPT. 126.—**Pitch of a tuning-fork (method 2).** Stretch on a sonometer a fine steel wire, attaching a weight sufficiently great to give with a length of at least 40 cm. a fundamental note in unison with that of the fork. Accurately adjust the movable bridge so that no *beats* can be observed when the string and fork are vibrating, the stem of the fork being pressed against the board of the sonometer. The absence of beats is detected readily when one end of a short wooden rod (or the cardboard case of a thermometer) is pressed against the sonometer board, and the ear pressed against the other end of the rod.

Measure accurately the length of wire between the two bridges, and note also the weight attached to the wire. With a fine-edged file make two shallow file marks on the wire, the marks being at least 30 cm. apart. Measure the distance between the file marks, unload the wire, and with wire pliers cut the wire at the file marks. Weigh accurately this known length of the stretched wire, and calculate the weight of unit length. Apply the data thus obtained to the equation

$$n = \frac{1}{2l}\sqrt{\frac{\mathbf{F}}{m}},$$

in order to calculate the pitch of the wire and fork.

EXAMPLE.—The same c' tuning-fork was used as in Expt. 122.

Size of steel wire used, No. 32, S.W.G.
Length of wire, in unison with tuning-fork, 58·35 cm.
Stretching force (**F**), 6000 gm.
Length of wire weighed, 57·6 cm.
Weight of this wire, 0·3552 gm.
Weight (m) of unit length, 0·00617 gm.

Hence,

$$n = \frac{1}{116 \cdot 7}\sqrt{\frac{6000 \times 981}{0 \cdot 00617}} = 265 \cdot 0 \text{ vibrations per sec.}$$

EXPT. 127.—**Velocity of sound in air, by resonance.** Support a glass tube T (Fig. 111), about 20 cms. by 3 cms., open at both ends, in a vertical position with its lower end dipping into water contained in a wider cylinder C. Hold over the upper end of tube T a vibrating tuning-fork F, of which the rate of vibration is known; and adjust the position of T so that the greatest reinforcement of the sound is obtained. Measure the distance from the top of T to the water level. Put T out of adjustment, and repeat the observation at least four times, and take the mean of these results.[1]

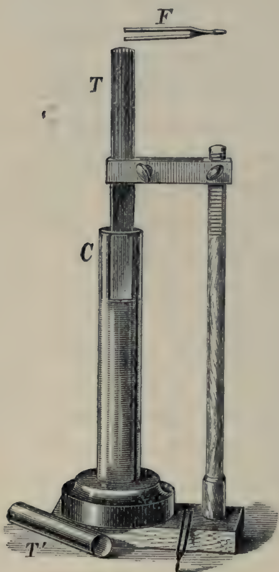


FIG. 111.—Resonant air column.

In any vibrating system the length of one complete wave is equal to *four* times the distance between any *node* and the nearest *antinode*. In the above experiment, the water surface within the tube can only be a node; similarly, the open end of the tube can only be an antinode. Hence, if the length of the air column, as measured above, be l, the wave-length of the note emitted is $4l$. Also, the velocity of transmission through any medium is equal to the wave-length multiplied by the total number n of waves emitted in each second; hence, if n be

[1] Strictly speaking, the position of the antinode is slightly *outside* the end of the tube; and this distance depends on the diameter of the tube. A more exact measure of the quarter wave-length is obtained by adding 0·8 of the radius of the tube to the length measured from the water surface to the top of the tube.

the frequency of the fork used in the above experiment, the velocity v of sound in air, at the temperature of the room, is given by the equation $v = n \times 4l$. Calculate the value of v.

Note the temperature of the room, and calculate the theoretical value from the formula $v = (33200 + 60t)$ cm.; where t is the temperature measured on the Centigrade scale.

EXPT. 128.—**Velocity of sound in wood or glass (by sonometer and tuning-fork).** Adjust a sonometer string until it is in unison with a tuning-fork of known pitch. Measure the length of the string. Firmly hold at its middle point a round rod of mahogany (oak or pine) about 6 ft. long, and set up in it longitudinal vibrations by rubbing it with a resined leather. Adjust the length of the sonometer string to unison, and measure this length. Calculate from these measurements the pitch of the note emitted by the rod. Measure the length of the rod.

As the middle point of the rod is fixed, it must be a node; and *each* end of the rod is an antinode. Hence half the length of the rod must be equal to one-quarter of the wave-length of the sound waves in the wood; the wave-length in the wood is equal, therefore, to *twice* the length of the rod. The velocity of the waves in the wood will be given by the product

$$(\text{pitch of note}) \times (\text{twice the rod's length}).$$

In a similar experiment with a long glass rod, the vibrations are more readily set up by using a wet rag instead of resined leather.

MAGNETISM.

CHAPTER XVI.

MAGNETISATION OF STEEL AND IRON.

EXPT. 129.—**Magnetisation by the method of "single touch."** Fix a darning-needle [1] to the table by pellets of soft wax, and stroke the needle from eye to point with the N.-seeking pole of a bar-magnet, lift the pole from the needle, and bring it down again on to the eye of the needle, and repeat the stroking several times. Test the needle for polarity at its ends (i) by dipping the ends into iron filings, and (ii) by suspending [2] it so that it swings freely in a horizontal plane. Note which end of the needle is N.-seeking. Magnetise

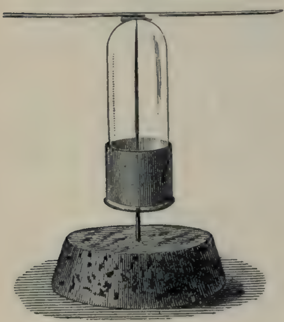


FIG. 112.—A method of supporting a magnetised needle.

[1] A short piece of clock-spring, with one end marked, is an excellent substitute for a needle.

[2] Small magnets, such as needles, may be suspended in a paper stirrup, or bent copper wire, attached to a single silk fibre. A *bundle* of unspun silk fibre should be used for heavy bar-magnets. An inverted test-tube, 2 inches long, pivoted on the point of a vertical needle (Fig. 112), is the most efficient support for small magnets: the magnet is fixed by soft wax to the top of the tube, and stability is ensured by placing a narrow collar of sheet-lead round the rim of the tube.

another needle, but stroke it from point to eye, using as before the N.-seeking pole of the bar-magnet. Note the difference in the polarity of the two needles, and state a rule as to the polarity obtained by a given procedure.

To test whether the magnetisation is permanent, drop the needle repeatedly on to the floor, or strike it with a *wooden* mallet, and afterwards repeat the tests for polarity.

With crucible tongs hold the needle in a Bunsen flame, and heat it to bright redness; when the needle is cool, repeat the tests for polarity.

Magnetise in the same way a piece of *soft iron*: either a long wire nail or a narrow strip cut from a biscuit-tin is suitable. Note more particularly any difference, between steel and soft iron, in the permanence of the polarity.

EXPT. 130.—**Magnetisation by means of an electric current.** Wrap a spiral of cotton-covered copper wire round a piece of thin-walled glass tubing (about 5 inches long and 0·2 inch bore). Place inside the tube an unmagnetised needle or piece of clock-spring with one end marked. Include the spiral in an electric circuit consisting of a battery, a rheostat (for regulating the strength of the current), and an ammeter (p. 206). Allow a fairly strong current to pass for a few seconds, tapping the spiral on the table during the process.

Before removing the needle from the coil, start from the battery to trace the direction of the current round the spiral (see p. 203), and apply the following rule: *When the spiral is held end-on to the eye of the observer, and the apparent direction of the current round the spiral is clockwise, the end of the spiral near to the eye possesses, and will impart to steel*

within the spiral, S.-seeking polarity. When the coil is viewed in the opposite direction, the direction of the current will appear to be anti-clockwise, and this end will impart N.-seeking polarity. (Note the direction of the arrows on the letters N and S in Fig. 113.)

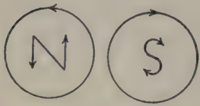


FIG. 113.

Remove the needle from the coil, test its polarity, and note whether it is in accordance with the rule previously stated.

EXPT. 131.—**Localising the poles of a bar-magnet.** (i) Place a long bar-magnet on a sheet of paper stretched on a drawing-board, and mark its position by passing

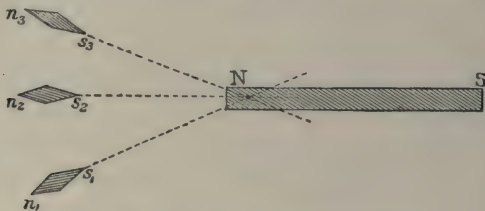


FIG. 114.—Locating the poles of a bar-magnet.

the point of a pencil round its edges (Fig. 114). Place a compass-needle at n_1s_1, and move the board round so that the needle is pointing directly towards the north: this prevents the earth's magnetic field from influencing the direction of the needle. Mark on the paper the direction in which the needle points. Repeat this, with the needle in two other positions, such as n_2s_2 and n_3s_3. Remove the magnet, and produce by means of a straight edge the three directions obtained. The point of intersection of these lines indicates the position of the pole of the magnet. Locate in the same manner the other pole of the magnet.

In short thick magnets (about 10 cm. long) the poles are situated about 1 cm. from each end. If the magnet is long, and only 1 or 2 millimetres wide, the poles approximately coincide with the ends. In the case of *Robison* magnets, which consist of straight needles terminating at each end in steel spheres, the poles will be found to be exactly at the centres of the spheres. The distance between the poles of a magnet is termed its *magnetic length.*

(ii) As an alternative method the following may be adopted: Place on a piece of squared paper a bar-magnet with its edge coinciding with one of the lines on the paper, and trace the outline of the magnet with a pencil. Adjust the paper and magnet so that its length points in an east-west direction. Place a compass-needle near the side of the magnet, and move it slowly along the paper and parallel to the magnet until it comes to a position where it points exactly north and south: the locating of this position is aided by the use of the squared paper. The needle is now pointing directly towards the pole of the magnet.

MAPS OF MAGNETIC FIELDS.

EXPT. 132.—**Map of the earth's magnetic field.** Fasten a square sheet of white paper (80 cm. × 60 cm.) on a table, with one edge pointing approximately north and south. Mark off one of the edges pointing east and west into spaces about 5 cm. wide. Place a sensitive compass-needle so that one of its poles is just over one of the marks, and indicate by means of a pencil mark the direction in which the other pole is pointing. Move the needle until its first pole is exactly over the second pencil mark; continue this process of marking the directions of the compass-needle until a series of marks have

been obtained completely across the paper. Join up these points by a continuous pencil line.

Plot other lines in a similar manner, in each case starting from one of the equidistant pencil marks at the edge of the paper. Indicate by means of arrow-heads the direction in which the north-seeking pole of the compass-needle *tends* to move; this is called *the positive direction of the magnetic field.*

EXPT. 133.—**Effect of a piece of soft iron on the earth's field.** Place a long strip of thick soft iron on a sheet of paper, its length coinciding with a north-south line. The iron must be entirely free from permanent polarity, and must be tested previously for this. By means of a compass-needle map the magnetic field in the region near to the iron, using the method of the preceding experiment.

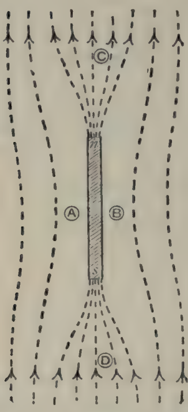
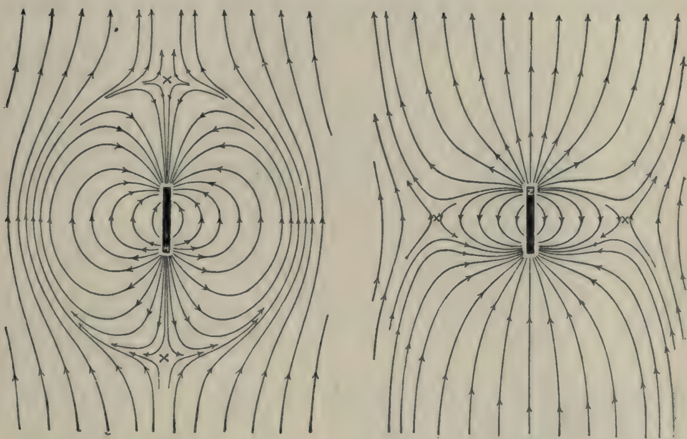


FIG. 115.—Effect of soft iron on the earth's field.

A map resembling Fig. 115 should be obtained. In the regions marked A and B the lines of force are less closely distributed, showing that the *intensity* of the field is there diminished; similarly, in the regions marked C and D, the intensity is increased. With a compass-needle placed near to the side of the soft iron, test the temporary polarity set up in its ends.

EXPT. 134.—**Resultant magnetic field due to a bar-magnet and the earth.** Fasten a sheet of paper on the table, as in Expt. 132. Determine the north and south line by means of a compass-needle, and place a small bar-magnet at the centre of the paper with its axis pointing

north and south, and its N.-seeking pole pointing towards the south. Starting from a series of points marked along the top edge of the paper, map the lines of force in the same way as before. To fill in the space near the magnet, draw a pencil line across the paper passing through the centre of the magnet and at right angles to



(i) (ii)

FIG. 116.—Map of the combined field due to the earth and a bar-magnet. (i) N. pole pointing south; (ii) N. pole pointing north.

its axis; use equidistant points on this line as starting points for tracing out additional lines of force. Fig. 116 (i) represents a map obtained in this manner.

Fig. 116 (ii) represents the map obtained when the N.-seeking pole of the magnet is pointing towards the north.

EXPT. 135.—**Iron-filing maps of magnetic fields.** Maps of the magnetic field *in the immediate neighbourhood* of one or more magnets can be obtained readily by means of iron filings. Lay a piece of *paraffined-paper* over the

magnet, or magnets, and support it horizontally by pieces of wood, of the same depth as the magnet, placed on either side. From a muslin bag, or a pepper-box, sprinkle filings uniformly over the paper, and tap the paper gently with a pencil point or a piece of stout copper wire, so as to give freedom of movement to the filings. Melt the wax slightly by passing a Bunsen flame over the paper, and allow it to cool.

Comparison of Magnetic Field Intensities.

The time of one complete oscillation of a suspended magnet is expressed by the equation

$$t = 2\pi\sqrt{\frac{I}{2lmH}},$$

where I is the "moment of inertia,"[1] $2l$ the magnetic length of the magnet, m the pole strength, and H the intensity of the field. If n is the number of oscillations of a suspended magnet described in a given time, then n is inversely proportional to t, and therefore *directly proportional to the square root of* H. Hence

H is proportional to n^2.

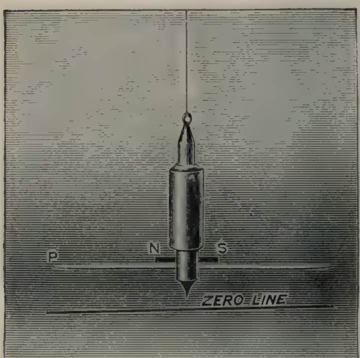


FIG. 117.—Vibration magnetometer.

Searle's *vibration magnetometer* (Fig. 117) is one form of suspended magnet suitable for the following experiments. It consists of a brass cylinder carrying a short magnet NS and an aluminium pointer P.

[1] See footnote 1, p. 182.

EXPT. 136.—**Comparison of the intensity at different parts of a room or building.** Suspend the apparatus just above the table, and lay a piece of paper immediately under the pointer. Bring the magnet to rest, and mark with a pencil line the position of rest of the pointer. Set the magnet vibrating through a *small* angle. Note the time when the pointer passes the position of rest, count the number of subsequent passages *in the same direction*, and when at least thirty complete oscillations have been described note again the time at the instant of passing the position of rest. Calculate the number (n_1) of oscillations which would be described in one minute. Repeat the observation at some other position in the room, and determine the number (n_2) of oscillations described in one minute. Then, if H_1 and H_2 are the intensities of the magnetic fields in the two positions,

$$H_1/H_2 = n_1^2/n_2^2.$$

EXPT. 137.—**Effect of soft iron on the intensity of the earth's field.** Suspend a vibration-magnetometer in one of the positions used in the previous experiment, and lay on the table a long strip of soft iron with its axis in the north-south line passing through the centre of the suspended magnet, and its near end 5-10 cm. from the magnet. Determine, as before, the number of oscillations described in one minute, and compare the intensity of the field with that of the field before the iron was placed in position. Carry out this experiment (i) with the iron to the north of the needle, and (ii) with the iron to the south of the needle, the distance of the iron being the same as before. These observations correspond to the positions C and D of Fig. 115.

Similar observations may be made when the magnetometer occupies positions (A and B) on each side of the iron.

Comparison of Moments of Bar-Magnets.

The *moment* (M) of a bar-magnet is defined as the product of the pole-strength (m) and the magnetic length ($2l$) of the magnet; hence

$$M = 2lm.$$

The intensity of field, due to a bar-magnet, at a point on the axis of the magnet produced.—Let NS (Fig. 118) represent a bar-magnet of pole-strength m and magnetic length $2l$.

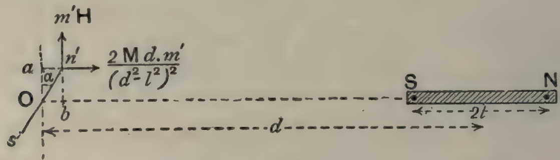


Fig. 118.—Deflection of a compass-needle by a bar-magnet.

Let O be a point on the axis of the magnet produced, and distant d cm. from the centre of the magnet. The intensity of the field at O, due to the pole S, is $m/(d-l)^2$; and that due to N is $-m/(d+l)^2$; hence the **resultant intensity** at O is

$$\frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} = \frac{m \cdot 4ld}{(d^2-l^2)^2}$$
$$= \frac{2Md}{(d^2-l^2)^2}.$$

When the magnet NS is placed in an east-west position, and a small compass-needle ($n's'$), of pole-strength m', is placed at the point O, the force acting on each pole of $n's'$ tending to deflect it will be $2Md \cdot m'/(d^2-l^2)^2$; and the force acting on each pole tending to restore it to its normal direction will be $m'H$, where H is the horizontal intensity of the

earth's field. The position of equilibrium of the compass-needle will be such that the moments of these forces round O will be numerically equal, or

$$\frac{2Md.m'}{(d^2-l^2)^2} \times Oa = m'H \times Ob,$$

or

$$\frac{M}{H} = \frac{(d^2-l^2)^2}{2d} \times \frac{Ob}{Oa}$$

$$= \frac{(d^2-l^2)^2}{2d} \times \tan \alpha,$$

where α is the angle of deflection of the compass-needle.

The comparison of the moments of two magnets requires the determination of the ratio M/H for each, and the ratio of the values obtained gives the ratios of the moments. The observations are made with a *deflection-magnetometer*, which consists of a compass-needle, with long pointer travelling over a circular scale; and, for convenience in measuring the distance *d*, two straight wooden scales are fixed to the body of the instrument at opposite ends of a diameter, the scales measuring distances from the centre of the compass-needle.

EXPT. 138.—**Comparison of the moments of two bar-magnets.** Adjust the magnetometer so that its arms are perpendicular to the magnetic meridian. Place the bar-magnet (marked A) on the right-hand arm of the magnetometer, and with its centre 25 cm. away from the compass-needle. Note the reading of each end of the pointer. (*N.B.*—Tap the instrument before taking this and subsequent readings.) Reverse the magnet and again take the readings. Repeat all these readings with the magnet transferred to the left-hand arm of the magnetometer. Find the mean value (α) of the deflection, and calculate the value of the ratio M_A/H from the equation

$$M_A/H = (d^2 - l^2)^2 . \tan \alpha / 2d.$$

As a check on the accuracy of the observations and calculation it is advisable to repeat the observations, increasing the distance (a) to 30 cm.

Obtain a similar series of observations with the other magnet (marked B), and calculate the ratio M_A/M_B.

When a magnet is swinging freely, the time of one complete oscillation is given by the equation

$$t=2\pi\sqrt{\frac{I}{2mlH}}=2\pi\sqrt{\frac{I}{MH}},$$

where M is the moment of the magnet, H the horizontal intensity of the field in which the magnet is swinging, and I is the *moment of inertia*[1] of the magnet. The above equation may be written $MH=4\pi^2I/t^2$, and the moments of two magnets may be compared by determining the ratio of the value of MH found separately for each magnet.

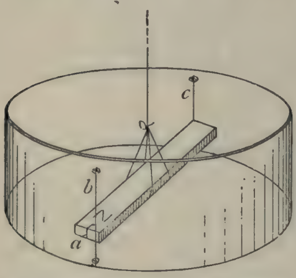


FIG. 119.—A method of suspending bar-magnets.

Fig. 119 suggests a convenient arrangement for finding the time of oscillation of a magnet. A length of *unspun* silk fibre, doubled back twice upon itself and knotted, makes the best suspension. A piece of white cotton is stretched vertically down one of the end-faces of the magnet, and held in position by small pellets of soft wax, as shown at *a*. Two other pieces of white cotton, *b* and *c*, are fixed vertically and diametrically opposite on the outside of a shallow circular glass dish.

[1] The moment of inertia can be calculated from the dimensions and weight of the magnet. In the case of a rectangular bar it is expressed by the equation

$$I=\left(\frac{(\text{length})^2+(\text{breadth})^2}{12}\right)\times\text{weight}.$$

EXPT. 139.—**Comparison of the moments of two magnets by the method of vibration.** Measure the dimensions, and find the weight of two magnets (marked A and B; if possible, use the same pair of magnets as in Expt. 138). Calculate the moments of inertia, I_A and I_B, of the magnets. Suspend one of the magnets inside the dish, and adjust the position of the dish so that the three threads are in the same straight line when the magnet is at rest. In order to screen the magnet completely from air currents, cover the dish with a sheet of stiff paper in which a circular hole has been cut at the centre, and a slit cut from the hole to one edge. By means of a distant magnet, set the suspended magnet oscillating through a *small* angle. View the apparatus with one eye, horizontally, and placed so that the threads *b* and *c* coincide. When the thread *a* passes thread *b* start a stop-watch, and count subsequent passages, in the same direction, of thread *a* until fifty complete oscillations have been completed; stop the watch and calculate the time of one complete oscillation. Finally, calculate the value of MH. Similarly determine the value of MH for the second magnet, and find the ratio of the moments of the two magnets.

EXPT. 140.—**Approximate value of the earth's horizontal intensity.** When the same magnet has been used for Expts. 138 and 139, thus obtaining for it the value of both M/H and MH, the results can be used for calculating the value of H, since

$$\frac{MH}{M/H} = H^2.$$

STATIC ELECTRICITY.

CHAPTER XVII.

FUNDAMENTAL PHENOMENA.

EXPT. 141.—**Attraction and repulsion of charged bodies.** (i) Suspend horizontally a glass rod (or tube) which has been rubbed previously with silk, quickly bring near to the rubbed end of this rod another glass rod which also has been rubbed with silk. Notice the *repulsion.*[1] Observe also the mutual repulsion between two rods of vulcanite which have been rubbed with fur or flannel. Fig. 120 suggests a convenient method of suspending rods; it is made of thick copper wire, and is suspended by a bundle of unspun silk fibres.

FIG. 120.—A suspension for electrified rods.

(ii) Suspend a rod of vulcanite rubbed with fur (or flannel), and bring near to it a rod of glass rubbed with silk. Notice the *attraction.*

[1] Glass surfaces are often peculiarly damp; and, when so, they will not retain a charge of electricity. Rods therefore should be warmed and dried either in a hot oven or by laying them on a hot sand-bath. Rods of fused silica are a very efficient substitute for glass, and should be used when available; no previous drying is necessary.

The universally recognised terminology states that glass rubbed with silk acquires a **positive**[1] charge, and that vulcanite rubbed with fur acquires a **negative** charge. The above observations demonstrate that **bodies with like charges repel, and bodies with unlike charges attract, one another.**

(iii) Suspend a charged vulcanite rod, and hold the hand near to it. Observe the *attraction.* Observe also that the hand will attract a charged glass rod. Evidently, therefore, **an uncharged body attracts a charged body,** whatever the sign of the charge.

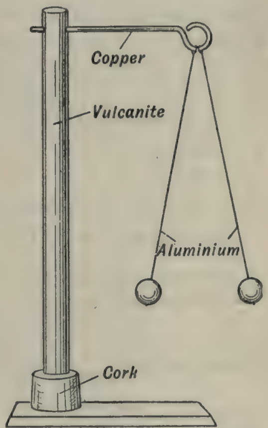


FIG. 121.—A pith-ball electroscope.

The above results can be observed more satisfactorily with the aid of a **pith-ball electroscope** (Fig. 121); in the instrument represented, the balls of pith are suspended by very thin aluminium wire from a horizontal arm of thick metal wire, which is supported by means of a vulcanite rod.

EXPT. 142.—**Use of a pith-ball electroscope.** Charge a vulcanite rod, and bring it near to the pith-balls. Observe how the balls are attracted to the rod and cling to its surface; in a few moments they will have acquired some of the rod's charge, and they will be repelled; also, since the balls have like charges, they also repel one another. Verify that a neighbouring charge, if negative, repels the balls; but it attracts them if positive. This indicates

[1] When a rod of glass (or of silica) has been heated *in a flame* it acquires a *negative* charge when rubbed with silk. This abnormal behaviour is most readily removed by passing the rod through the closed hand. On account of this peculiar property it is necessary to avoid using the direct heat of a flame for warming the rods. Even a hot furnace has the same effect on glass rods.

how the instrument is used for the purpose of verifying the sign of a neighbouring charge.

Observe also that *the hand will attract the charged pith-balls, whatever the sign of the charge on the balls.* Hence, if when using the instrument to verify a neighbouring charge an attraction of the balls is observed, the neighbouring body may be (i) either charged oppositely, or (ii) uncharged. In such a case, the charge on the pith-balls should be reversed, after which the neighbouring body (if charged) will repel the balls.

EXPT. 143.—**Simultaneous production of both kinds of electrification.** This experiment serves to demonstrate that when two surfaces are rubbed together, opposite charges are generated in equal quantity. Fig. 122 suggests alternative methods. In one of these, a polished vulcanite rod has a loose-fitting cap of flannel (like a long finger-stall). One end of a strong *silk* thread is knotted to the flannel and wound several times round it. After pulling the thread until it is all unwound, hold the rod (with the cap still on) near to a charged electroscope; the absence of any effect will suggest either an absence of charge or an equality of opposite charges. By means of the silk thread remove the cap, and verify (i) the negative charge on the rod and (ii) the positive charge on the flannel.

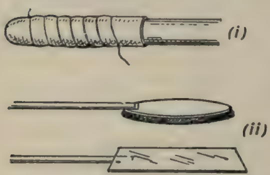


FIG. 122.—Expt. 143.

In the other form of apparatus a small square of glass and a cardboard disc with a silk pad attached are each fixed to a vulcanite handle. Warm the glass, vigorously stroke it with the silk pad, and carry out observations as described previously.

EXPT. 144.—**Conductors and insulators.** Charge an electroscope. Hold in the hand a short length of dry cotton thread, and allow the free end to touch the horizontal metal arm of the electroscope. Observe that the charge slowly, but distinctly, escapes along the thread. Recharge the electroscope and repeat the observation, using the following substances in sequence: Dry silk thread, damp silk thread, warm dry glass rod, damp cold glass rod, narrow strip of paper, any metal wire, sealing wax, paraffin wax, roll sulphur, the hand, charcoal, fused silica, a piece of broken china, and any kind of wood.

Arrange these materials in groups as (i) good conductors, (ii) poor conductors, or (iii) insulators.

A flame, such as that of a Bunsen burner or of a spirit lamp, is an excellent conductor. Test this by passing through the flame a charged vulcanite rod.

EXPT. 145.—**Electrification of metals.** Any metal rod or plate held in the hand cannot be charged by rubbing with silk or fur, since any charge acquired instantaneously escapes through the metal and the hand, which are both good conductors. But if a sheet of metal is supported on a vulcanite handle, and then *flicked* (not rubbed) with a piece of fur, it rapidly acquires a *negative* charge. Verify this by means of a charged electroscope.

ELECTROSTATIC INDUCTION.

Potential.—*The electric potential at any point in an electric field of force is measured by the mechanical work required to convey a unit +ve charge from an infinite distance up to the point.* The unit charge then possesses **potential energy**

equivalent in amount to the mechanical work done on it: this amount of energy represents the so-called **electric potential** at the point referred to. Hence, in the field surrounding a fixed +ve charge, the potential at all points is *positive* (assuming that no other charges are in the neighbourhood); and this potential increases as the fixed charge is approached. At an infinite distance the potential is zero. When the fixed charge is −ve, work must be done in *withdrawing* the unit +ve charge from the neighbourhood of the fixed charge, and the maximum of work would be required if initially the unit charge is at a point almost touching the fixed charge. Hence the potential is least at points near to the fixed charge, and gradually increases as the distance increases, finally becoming zero at an infinite distance. The field round a fixed −ve charge is said to be one of *negative* potential.

The sign (*i.e.* whether +ve or −ve) and magnitude of the potential at a series of points in an electric field can be represented graphically by taking a straight line to represent zero potential, and by drawing a perpendicular line from each point; +ve potential is represented by a line drawn *above* the zero line, and −ve potential by a line drawn *below* the zero line, the length of the perpendicular in each case representing the magnitude of the potential at the point. The slope of the line joining the extremities of the perpendiculars represents the variation of potential along the base line. Examples of such diagrams are shown in Fig. 123. If accurately drawn, the lines indicating fall of potential in many cases should be curved, and not straight; this applies especially to the field of force between two *small* charged conductors; the straight lines in Fig. 123 are uniformly adopted only for simplicity.

The simplest method of deducing the sign of the potential at any point is as follows: Imagine that a small insulated and uncharged conductor is placed at the point, and consider what will happen when the conductor is momentarily touched by the finger. The conductor, of course, has a potential the same as that of the region

where it is situated. If a positive charge passes from the conductor to the earth, the conductor must have had *positive* potential relatively to that of the earth; but if a positive charge passes from the earth to the conductor, the latter must have had *negative* potential. This reasoning is based upon the fundamental principle that a positive charge always tends to pass from a point of higher potential to a point of lower potential.

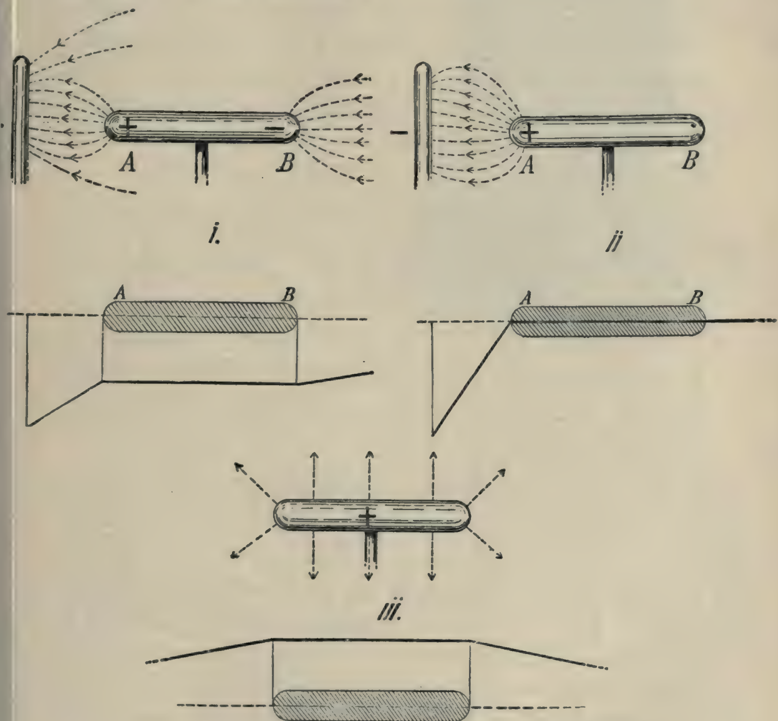


FIG. 123.—Charging an insulated conductor by electrostatic induction.

EXPT. 146.—**Electrostatic induction.** (i) Hold a −ly charged vulcanite rod near one end of a conducting

cylinder mounted on an insulating stand (Fig. 123, i). Hold a proof plane[1] with its flat side in contact with the end A of the cylinder. By means of a charged electroscope verify that the proof plane has a + ve charge. Verify, in the same way, that the end B has a — ve charge. Keeping the charged vulcanite rod still in the same position, momentarily touch the cylinder *at any part of its surface* with the finger; verify that the end A still has a + ve charge and that the end B now has no charge (Fig. 123, ii). Remove the vulcanite rod to a distance, and verify the + ve charge, which is distributed over all parts of the conducting cylinder (Fig. 123, iii).

Make diagrams, as in Fig. 123, for the three stages of this experiment. Also, below each sketch make a *potential diagram* showing the changes of potential along a line coinciding with the axis of the cylinder.

(ii) Using, instead of the charged vulcanite rod, a + ly charged glass rod, repeat all the above observations, and verify finally that the cylinder is now charged *negatively*.

Make three separate sketches to represent each stage of the experiment, and, if possible, add to each a potential diagram.

EXPT. 147.—**Theory of the gold-leaf electroscope** (Fig. 124). (i) Hold a — ly charged rod of vulcanite over the disc. The leaves are at a higher potential than the disc, consequently + ve electrification passes from the leaves to the disc, giving the former a — ve and the latter a + ve charge.

[1] A small copper coin fixed on the end of a rod of sealing wax is an efficient "proof plane."

The charge on the leaves induces a +ve charge on the tinfoil. Lines of force (Fig. 125, i) proceed across from each tinfoil strip to the nearest metal leaf, resulting in the leaves being pulled apart. The same number of lines of force also pass from the disc to the vulcanite. The degree of divergence will depend upon the number of lines of force passing between the leaves and the tinfoil.

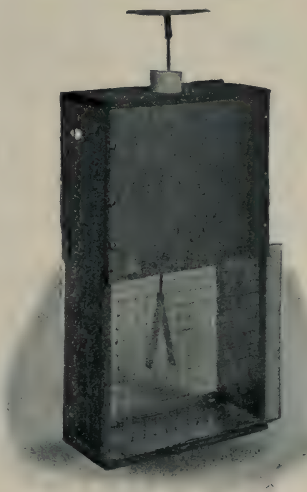


FIG. 124.—A gold-leaf electroscope.

(ii) Hold the vulcanite still in the same position, and touch the disc with the finger. The potential of the leaves is raised to zero, the lines of force between the tinfoil and the leaves disappear, and the leaves collapse (Fig. 125, ii).

(iii) Remove the vulcanite to a distance. The +ve charge distributes itself uniformly over the conductor, a portion going into the leaves and inducing a −ve charge on the tinfoil. The lines of force thus brought into play cause the leaves to diverge (Fig. 125, iii). *The electroscope has been charged +ly by induction.*

(iv) Hold a +ly charged glass rod over the disc. The potential of the disc is raised above that of the leaves. The +ve charge in the leaves increases, and the increased number of lines of force causes the leaves to diverge more (Fig. 125, iv).

(v) Hold a —ly charged rod of vulcanite over the disc. The potential of the disc is lower than that of the leaves. Part of the +ve charge in the leaves passes to the disc, thus diminishing the number of lines of force between the leaves and the tinfoil (Fig. 125, v).

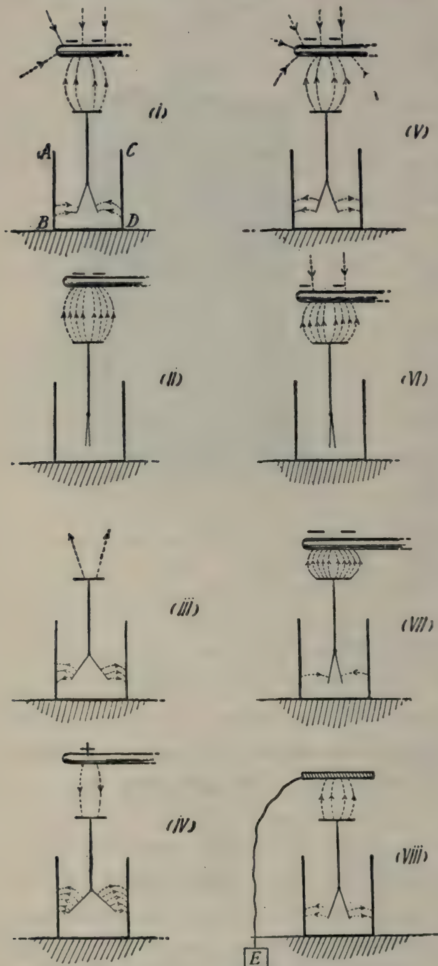


FIG. 125.—A gold-leaf electroscope under various electrical conditions.

(vi) Hold the vulcanite nearer the disc. The potential of the disc is still further diminished, more +ve charge passes from the leaves to the disc, and the divergence of the leaves is diminished. If the potential is reduced to zero, the leaves will have no divergence (Fig. 125, vi).

(vii) Hold the vulcanite still nearer to the disc. The potential of the disc is now negative, and the leaves will diverge with —ve electricity (Fig. 125, vii).

(viii) Remove the vulcanite to a distance, and hold the hand, or some other earth-connected conductor, over the disc. The +ve charge on the disc now induces a −ve charge on the under side of the hand, resulting in some (or all) of the lines of force being transferred from the leaves to the disc, and so reducing the divergence of the leaves. The nearer the hand is to the disc the greater is the reduction in the divergence of the leaves (Fig. 125, viii). The same result may be expressed by saying that the induced −ve charge on the hand creates a region of −ve potential round the disc, and consequently lowers the potential of the instrument.

(ix) Repeat (i) and (ii), but use a +ly charged glass rod instead of the vulcanite. The electroscope will now be charged −ly. A +ly charged body held over the disc will now *diminish* the divergence, and a −ly charged body will *increase* the divergence. An earth-connected conductor held over the disc will *diminish* the divergence (as when the instrument is +ly charged).

(x) Place the uncharged electroscope on an insulating stand.[1] Connect the disc to the tinfoil base by means of a thin wire. Hold a charged body near to the instrument, and observe that the leaves do not diverge. The leaves and the tinfoil are at the same potential, and therefore no lines of force pass between them to cause a divergence.

Remove the thin wire, and observe that a charged body near to the instrument will cause a divergence of the leaves. If the charged body is an electrified glass rod, the potential of the leaves and of the tinfoil will be raised, but not to the same degree; hence there will

[1] Flat slabs of paraffin wax serve the purpose admirably.

be a potential difference causing a divergence of the leaves.

Touch the tinfoil with the finger, so as to reduce its potential to zero. The potential difference is now greater, and this is shown by the increased divergence.

From these results we see that the divergence of the leaves depends upon the potential difference between the leaves and the earth-connected tinfoil; if the instrument is +ly charged, a *rise* in the potential of the leaves will produce an *increased* divergence; a *fall* in potential will produce a *reduced* divergence. The electroscope may therefore be used as a means of detecting any changes in potential.

This principle may also be applied in order to determine the kind of electrification on a body which is held over the disc, for if the body be +ly charged the potential of the electroscope is raised, and if the body be −ly charged the potential of the electroscope is lowered. The rules to observe will be as follows:

Electroscope charged +ly.
- **Increased divergence** *implies* **+ve charge.**
- **Diminished divergence** *implies* **−ve charge (*or* an earth-connected conductor).**

Electroscope charged −ly.
- **Increased divergence** *implies* **−ve charge.**
- **Diminished divergence** *implies* **+ve charge (*or* an earth-connected conductor).**

From these results it is evident that an *increased divergence* is the only sure test of a neighbouring charge. Similarly, it has been proved that in the case of a pith-ball electroscope (p. 185), *repulsion* of the balls is the only sure test.

EXPT. 148.—**Faraday's ice-pail experiment.** (i) Place a small can on the table. Introduce an insulated

sphere,[1] which is charged +ly, well inside the can, taking care that the sphere does not touch the can. Touch the inside of the can with a proof plane, and withdraw it, being careful not to touch the sphere or the edge of the can. Test the charge on the proof plane by means of an electroscope. The inside of the can has an induced −ve charge.

(ii) Place a small can on an insulating stand, and connect the can to the disc of an electroscope. Intro-

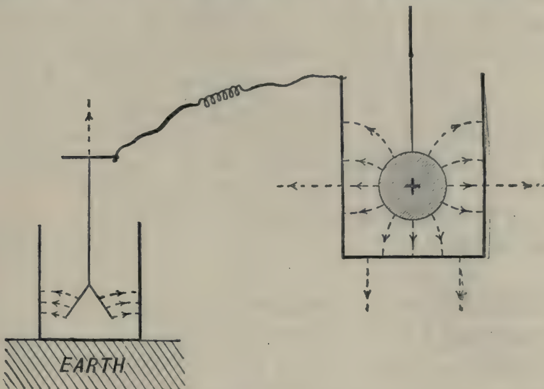


Fig. 126.—Faraday's ice-pail experiment.

duce an insulated sphere (or foil-covered bottle), which is charged +ly, well inside the can, taking care not to touch the sides of the can (Fig. 126). If the can be sufficiently deep, all the lines of force from the charged sphere are now intercepted by the sides of the can, on the inside of which an induced −ve charge is found.

The induced +ve charge is distributed partly over the outer surface of the can and partly over the

[1] Instead of the sphere, a convenient carrier may be made by covering the outside of a 2 oz. glass bottle with tinfoil, and fastening a rod of vulcanite to the cork to serve as an insulating handle.

electroscope, from both of which there are lines of force proceeding to any neighbouring earth-connected conductors. Observe the divergence of the leaves. Allow the sphere to touch the inside of the can, thus making it a portion of the inside of the charged conductor. The divergence remains unaltered. Remove the sphere, and test whether it still has any charge.

The sphere has no remaining charge, nor is there any charge on the inside of the now empty can. The sphere's charge has been exactly neutralised by the induced −ve charge on the can, without leaving an excess of either (for had there been an excess of either it must have proceeded to the outside of the can, and so modified the divergence of the leaves). All the lines of force which originally proceeded from the sphere evidently disappeared when the sphere touched the can.

Hence, **the total induced charge is equal and opposite to the inducing charge.**

EXPT. 149.—**The simple condenser.** (i) Charge an electroscope +ly. Observe the divergence of the leaves. Hold the hand just above the disc, and observe how the divergence is less than before. When the hand is removed the divergence increases to its original value. Evidently the "capacity" of the conductor was increased by holding the hand over the disc.

(ii) Hold an *insulated* metal plate over the disc of a +ly charged electroscope. The −ve charge induced on the lower surface of the plane tends to *lower* the potential in the region round the electroscope, while the +ve charge induced on the upper surface tends to *raise* the potential. Since the former is nearer to the electroscope than the latter, the *diminution* will slightly exceed the *increase*. Hence the potential of the electroscope is lowered very

slightly. Observe the slight diminution in the divergence of the leaves.

(iii) *Touch the metal plate* while still holding it in the same position. The divergence of the leaves is reduced still more. The induced +ve charge has disappeared, and no longer tends to raise the potential. The result is exactly the same as holding the hand at the same distance above the disc of the electroscope.

(iv) Charge an insulated sphere, which is connected to an electroscope by means of a long thin wire. Hold in the hand a disc of metal, much smaller in diameter than the sphere, about 3 or 4 cm. away from the sphere. Observe the diminution of divergence. Remove the small disc, and hold a much larger disc at the same distance as before. The divergence is diminished much more than before. Hence the larger disc has caused a greater diminution of potential—or, in other words, the capacity of the condenser is much greater when the large disc is used.

Vary the distance of the disc from the sphere, and note the effect of this upon the potential of the sphere.

EXPT. 150.—**The simple condenser (continued).** Fig. 127 represents a useful model for demonstrating the main principles of the condenser.

(i) Connect plate A to an electroscope by means of a long thin wire, and connect B to earth. Charge the plate A. Observe the divergence when B is about 20 cm. distant from A. Slowly move B towards A, and observe the diminution of divergence. Slowly remove B, and observe the gradual increase of divergence. The plates A and B constitute a simple form of condenser, in which the capacity of A is raised artificially by the presence of B.

(ii) Place B about 3 cm. from A, and charge A. Carefully insert a square slab of paraffin wax (about 1 cm. thick, and about the same area as the plates) between A and B. Notice the diminution of divergence, and how it increases to its original value when the slab is removed. (The diminution will be comparatively small, and will require careful observation.) State how the paraffin-wax affects the *capacity* of the condenser.

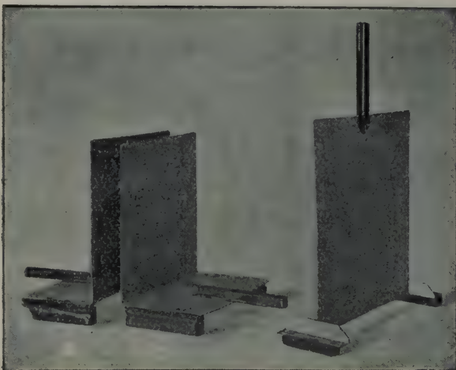


FIG. 127.—A simple condenser.

(iii) Give a +ve charge to plate A of the condenser, with the plates about 8 cm. apart. Connect the plate C (Fig. 127) to an electroscope by means of a thin wire; holding it by its insulating handle, place it between A and B, and parallel to both. Observe that the divergence is greater when C is nearer to A, and gradually diminishes as it is moved towards B. While C is in position, remove the connecting wire by means of an insulating handle, and verify that the charge on the electroscope is +ve. What information does this give as to the field of force between the plates?

Move C to a distance, and connect up to the electroscope again. While plate A remains charged, disconnect B from the earth and move it about 16 cm. away from A. Explore the field between A and B as before. The leaves diverge when C is near A, and

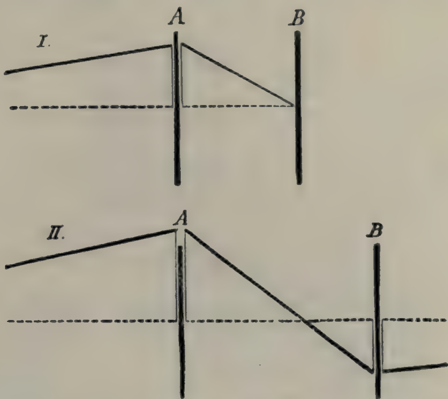


FIG. 128.—Potential diagrams of a simple condenser.

gradually collapse as C is moved towards B; when moved still nearer to B the leaves again diverge. Now remove the connecting wire, and verify the −ve charge in the electroscope, showing that the region near to B is one of −ve potential.

From these observations verify the potential diagrams shown in Fig. 128, i and ii.

VOLTAIC ELECTRICITY.

CHAPTER XVIII.

THE ELECTRIC CURRENT AND ITS EFFECTS.

Definition of units of measurement.—The following definitions were adopted at an International Conference held in 1908:

(i) The international **ohm** is the resistance offered by a column of mercury at 0° C, weighing 14·4521 gm., of constant cross-section and 106·30 cm. long. The ohm is equal to 10^9 *absolute* units of resistance.

(ii) The international **ampere** is the steady current which, when passed through a solution of silver nitrate in water, deposits 0·001118 gm. of silver per second. The ampere is equal to 0·1 *absolute* unit of current.

(iii) The international **volt** is the steady electrical pressure which when applied to the ends of a conductor having a resistance of one ohm produces a current of one ampere. The volt is equal to 10^8 *absolute* units of electrical pressure.

Sources of electric current.—In experimental work the student will have available, as a rule, the following sources of electric current: (i) Leclanché cells, or dry cells, (ii) small portable accumulators, (iii) Daniell cells, and perhaps (iv) a "standard" cell (such as the Weston cadmium cell). The following remarks upon the use of these sources will assist the student to select the type of cell most appropriate to the experiment in hand.

(i) **Leclanché cells** (Fig. 129).—These are used when a small, but not uniform, current is required; thus it is quite appropriate in the measurement of resistance by

means of the metre bridge or the Post-office box. When considerable current is taken from one of these cells, its **electromotive force** rapidly diminishes, owing to **polarisation**; but when the circuit is broken and the cell allowed to rest, it rapidly recovers its original electromotive force. The same remarks apply to the so-called "dry cell," which chemically is the same as the Leclanché cell. The advantage of this type of cell is that when once set up it may be left for several weeks without any further attention. The electromotive force of each cell may be taken as 1·43 volts.

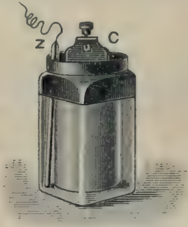


FIG. 129.—A Leclanché cell.

(ii) **Accumulators** (Fig. 130).—These are undoubtedly the most satisfactory source of a uniform current for experimental work. A small form of the cell, having a capacity of 4 ampere-hours,[1] can be purchased for about four shillings. Of course, the laboratory must have some means of recharging the accumulators periodically: this should be done regularly, at least once in every fourteen days, whether the cells are used or not. The chief disadvantage is that an accumulator is damaged seriously when an excessive current is taken from it; the maximum permissible current depends upon the size of the cell, and this should be marked on each one. *The current taken from a small portable cell should not exceed one ampere.* When they are to be used by inexperienced students, the cells should be enclosed in a wooden box with screw-down lid and external terminals; and a *fuse* (working at one ampere) should be fixed inside the box, and between the cell and the external terminals.

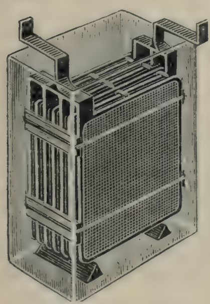


FIG. 130.—An accumulator.

[1] The *ampere-hour* is the quantity of electricity conveyed by a steady current of one ampere flowing for a period of one hour.

When fully charged the electromotive force of an accumulator is 2·2 volts, and when it falls to 1·8 volts the cell should be recharged before taking any further current from it. The liquid (dilute sulphuric acid) inside the cell slowly evaporates, and there is loss also by electrolysis during the charging; when the upper edges of the plates are exposed above the liquid, *distilled* water should be added in sufficient quantity just to cover the plates again.

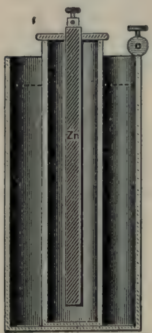


FIG. 131.—The Daniell cell.

The very small *internal resistance* of an accumulator is a particularly useful characteristic.

The Daniell cell (Fig. 131).—This cell is chiefly of historical interest; but in laboratories which do not possess any of the more recent standards of electromotive force, the Daniell cell can be used as a substitute. The great disadvantage of the cell is that it must be dismantled immediately the experiment is completed. The two liquids (dilute sulphuric acid and concentrated solution of copper sulphate) should be stored in separate bottles, the zinc rod washed, and the porous pot put to soak (not swilled) in a deep dish of clean water.

The E.M.F. of a Daniell cell is 1·08 volts, when the dilute acid is made by mixing 1 vol. of strong sulphuric acid with 12 vols. of water.

A standard cell (*e.g.* a Weston cadmium cell).—This should be used only by an experienced student. As such cells become polarised, even when traversed by a very small current, they should always be protected by connecting them to the experiment through a very high resistance (not less than 10,000 ohms). It is very seldom that a standard cell is required for any purpose other than the measurement of electromotive force by the potentiometer method. The E.M.F. is 1·018 volts approximately.

In the case of accumulators and standard cells, the poles

are always distinguished by the symbols + and − outside the cells. In other cases, in which a zinc rod or plate is used, it is sufficient to remember that the zinc is always the *negative* terminal.

Methods of connecting two, or more, cells.—When the negative pole of one cell is connected to the positive pole of the next, and so on, the cells are said to be connected *in series*; and they are then working *in conjunction.*

When one of two cells is reversed, so that the two negative poles are joined together, the cells are said to be *in opposition.*

When all the positive poles of two or more cells are joined together by thick wire, and all the negative poles similarly joined together by another wire, the group of cells may be regarded as equivalent to one large cell; and the cells are said to be connected *in parallel.*

Some simple appliances used in experimental electricity. (i) **A commutator** (Fig. 132).—Any device for readily reversing

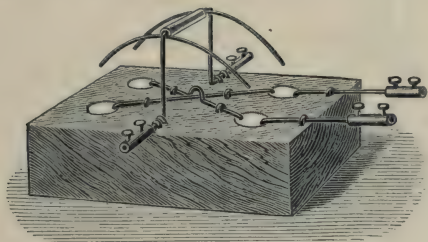


FIG. 132.—A commutator.

the direction of the current traversing a circuit is termed a commutator. The form shown in Fig. 132 can be made from the simplest materials; the wooden block has four circular holes containing mercury, and the mercury cups are connected diagonally by copper wire. Two other wires are fixed with their ends dipping into the mercury cups at one end of the block. The rocker is made from thick copper wire, and the two sides are separated by either glass tubing or ebonite, which serves to insulate one side from the other and also as a handle with which to move the rocker. When

in use, the source of electric current is connected to the fixed wires projecting beyond the end of the block; and the wires leading to the circuit are connected one to each side of the rocker. In the position of the rocker shown in Fig. 132 the circuit is broken (or, as it is sometimes expressed, *the circuit is open*). When the rocker is rotated to the right or to the left the circuit is closed, and the direction of the current in the circuit *beyond the commutator* depends upon which position has been given to the rocker.

(ii) **Resistance boxes and rheostats.**—For the comparison and measurement of unknown resistances it is necessary that other resistances of known magnitude be available. In a **resistance box**, a series of separate coils are arranged so that they may be used in any desired combination. Each coil consists of silk-covered wire wound on a cylinder; the free ends of the coil are connected to adjacent brass blocks *b* (Fig. 133) on the outside of the cover of the box; the space between each pair of blocks is occupied by a conical brass plug fitted with an ebonite handle *p*. When the plug is inserted, any current passing along the brass blocks is conveyed through the plug; but when the plug is removed, all the current has to pass through the resistance coils beneath. The resistance of each coil is marked, in *ohms*, on the lid of the box.

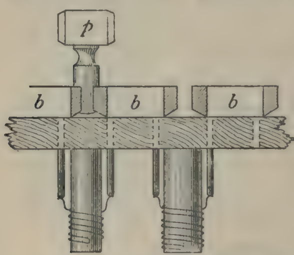


FIG. 133.—Construction of a resistance box.

In another type of resistance box, termed the "dial pattern," the coils are joined to metal studs arranged in a circle; one or more of the coils can be inserted into the circuit by rotating a metal arm pivoted at the centre of the circle.

It should be remembered always that it is very easy to damage a resistance box by sending through it too strong a current.

The **rheostat** may be described as an adjustable unknown resistance, and several different forms are available. Fig. 134

represents one in which a number of thin slabs of hard carbon are in contact, and the slabs can be squeezed together more or less tightly by means of a hand-screw. The more tightly the slabs are compressed, the less is the

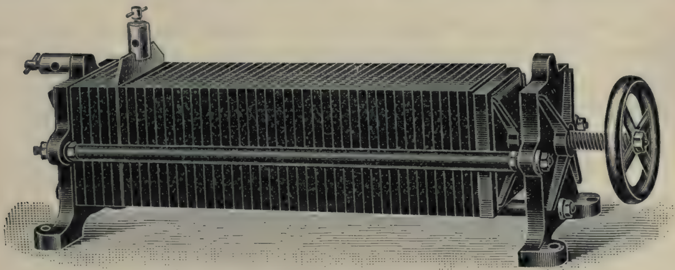


FIG. 134.—A carbon-block rheostat.

resistance offered by the slabs to a current passing through them. For general use this is the most efficient type of rheostat.

(iii) **Galvanometers, ammeters and voltmeters.**—Fig. 135 [1] represents one of the available types of simple portable galvanometer suitable for experiments, such as those with the metre-bridge, in which the *absence* of a current has to be detected. It consists of a small circular coil of wire (silk-covered); in the centre of the coil a small rectangular piece of cork is suspended by means of a silk fibre, and the cork carries six short pieces of magnetised steel needle. The pointer is made to occupy

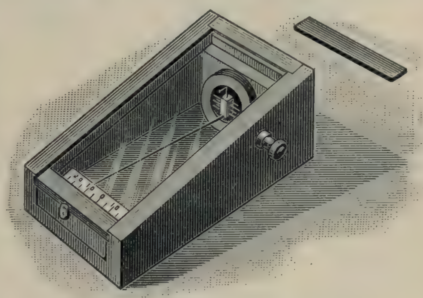


FIG. 135.—A simple galvanometer.

[1] The galvanometer shown in Fig. 135 is manufactured by Messrs. W. G. Pye & Co., Cambridge.

the zero position on the scale by means of a bar-magnet lying on the bench outside the case of the instrument. The same type of instrument may be used for the detection of weak currents, but it is not appropriate for quantitative measurement.

When greater accuracy is desired it is necessary to use a **mirror-galvanometer**, in which a beam of light is reflected from a small mirror attached to the suspended part of the instrument, the beam of light serving the purpose of the pointer used in the simpler instruments. In the **d'Arsonval galvanometer** (Fig. 136), which is the most suitable for general use, the magnets are fixed and the coil of wire is suspended—its principle therefore being the reverse of that of the type shown in Fig. 135. The coil is suspended by a fine strip of phosphor-bronze, which serves to conduct the current into the coil; and the current leaves the coil through a fine spiral spring. The instrument is adjusted so that the plane of the coil is parallel to the magnetic lines of force in the gap between the magnet poles. A current through the coil sets up opposite magnetic polarity at the opposite faces of the coil, as shown in Fig. 113 (p. 174), and the magnetic field causes a deflection of the coil, the direction of deflection depending upon the direction of the current. This instrument may be used for measuring extremely weak currents. A **tangent galvanometer** may also be used for measuring currents, as explained on p. 216. The principle of the instrument is described on pp. 211-14.

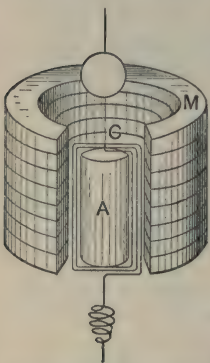


FIG. 136.—The d'Arsonval galvanometer.

When approximate accuracy of the measurement of a current is sufficient, it is more convenient to use some form of **ammeter**, many of which are identical in principle with the d'Arsonval galvanometer, except that only a fixed small fraction of the current passes through the suspended coil. Fig. 137 (i) represents the external view of one form

of ammeter; and Fig. 137 (ii) is a diagram of the main internal details. The permanent magnet NS is of the horse-shoe type; and a soft iron cylinder D, fixed between the poles of the magnet, serves to concentrate the lines of force within the space occupied by the pivoted coil C. The coil is wound on a rectangular frame of thin aluminium, it rotates on a horizontal axis, and the current is conducted into and from the coil by the fine springs s_1 and s_2. The current to be measured enters the instrument at the terminal

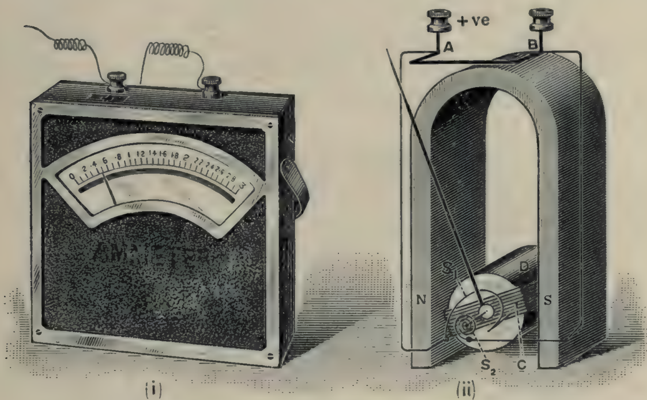


FIG. 137.—An ammeter (moving-coil type).

marked $+^{ve}$; at the point A the current divides; nearly the whole current passes to the point B through the short thick wire shown, while the remainder is conveyed to the coil through the spring s_1 and back through s_2 to the point B. The polarity set up in the coil causes it to rotate in a direction such that the pointer moves from left to right in front of the divided scale; and the rotation ceases when the magnetic forces are balanced by the mechanical forces set up in the springs. The relative resistances of the thick wire between A and B, and of the coil, are adjusted so that the scale reading indicates the strength of the total current. This construction makes the resistance of an ammeter very

small, and a very small portion of the energy of the current therefore is used up in the instrument.

A **voltmeter**, as its name indicates, is an instrument for measuring the difference of electrical pressure between the two points of a circuit to which it is connected. A frequent type of the instrument resembles closely the ammeter shown in Fig. 137, except that the thick wire between the points A and B is removed, and a high resistance is inserted between one of the terminals and the coil. This construction ensures that the current traversing the coil is very small, and not sufficient to disturb the distribution of electrical pressure along the main circuit. The coil is constructed, and the scale is graduated, so that the instrument indicates differences of electrical pressure in terms of the *volt.*

EXPT. 151.—**Polarisation in a voltaic cell.** (i) Fit up a simple voltaic cell consisting of a copper plate and a zinc plate immersed in *very* dilute sulphuric acid. Connect the cell in series with a tangent galvanometer (or a sensitive ammeter) and a resistance box. Adjust the resistance until a deflection of 50°–60° is obtained on the tangent galvanometer. Without altering any of the adjustments, remove the plates and dip the *copper* plate for a moment into a solution of copper sulphate ; replace the plates and read the deflection *as quickly as possible.* Note the deflection every half-minute for several minutes. (The film of copper sulphate solution delays the setting-up of polarisation, and allows the galvanometer-needle to become steady before the reduction of the current becomes evident.)

(ii) An alternative method of demonstrating polarisation in commercial cells is as follows : Connect together in series two Leclanché cells, but reverse the direction of one of them so that they tend to send current in opposite directions round the circuit, which should include

a resistance box and a tangent galvanometer. Notice that the deflection is very small or perhaps negligible. Temporarily break the circuit, and "short-circuit" one of the cells by connecting its poles with a short wire; allow this to remain for about five minutes, then re-establish the circuit as before. The cell which has been short-circuited will now exhibit marked polarisation, and the circuit will be traversed by a current due to the unpolarised cell. Note the deflection of the galvanometer every half-minute, and observe how the deflection gradually falls, showing that the polarised cell is recovering its normal condition.

Magnetic Effects of an Electric Current.

Expt. 152.—**Oersted's experiment.** (i) Connect the poles of an accumulator to a **commutator** or **current reverser**, the other terminals of which are connected through an adjustable resistance (or **rheostat**) to a long length of thin cotton-covered copper wire (Fig. 138). Stretch out a length of the wire, AB, so that it is horizontal and in the magnetic meridian, and

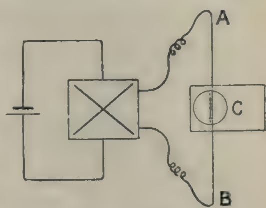


Fig. 138.

hold it immediately over a compass-needle (C). Now start the current by means of the commutator, and note the direction in which the current is passing, and also the direction in which the *north-seeking* pole of the compass-needle is deflected. Reverse the current by means of the commutator, and repeat the observations. Also, repeat the observations with the wire *below* the compass-needle.

Tabulate your observations in the following manner:

Position of Compass-needle.	Direction of Current.	Direction of Deflection of N.-seeking Pole of Needle.
Under the wire	N. to S.	
	S. to N.	
Over the wire	N. to S.	
	S. to N.	

Notice how all these results are included in the following rule (*Ampère's*): **Suppose a man to be swimming in the wire in the same direction as the current, and with his face towards the needle; the north-seeking pole is deflected in the direction of his left hand.**

(ii) It is evident that a current from N. to S. *under* the needle tends to deflect the needle in the same direction as when the current is from S. to N. and *over* the needle.

Hence the deflection observed in Expt. 152 (i) may be increased by doubling the wire over and under the needle. Verify this by wrapping the wire several times round the needle, and observe how the deflection increases.

To demonstrate that the magnetic lines of force, round a straight wire conveying a current, are circular and concentric with the wire requires a very strong current (15–20 amperes) if the field is to be mapped by iron filings. If this current is not available, much useful information can be obtained with a weak current and a compass-needle.

(iii) Lay a sheet of paraffined paper on a sheet of cardboard, and bore a *small* circular hole through the centre of both sheets. Clamp the cardboard and paper in a horizontal position, and thread a straight piece of thick copper wire (40 cm. long) vertically through the circular hole. Clamp the wire in this position, sprinkle iron filings on the paper, and connect the wire to a very large battery,

including a rheostat and an ammeter. Complete the circuit, and tap the cardboard. Break the circuit, and notice how the filings have arranged themselves in circles concentric with the wire (Fig. 139).

Fix the filings in position by warming the paper with a Bunsen flame. Place a compass-needle on the paraffined paper and near to the wire. Complete the circuit and observe the direction in which the needle points when placed to the north, south, east, and west of the wire. Reverse the direction of the current and notice that, in each position, the direction in which the needle points is reversed. Adjust the connections so that the current is passing *down* the wire, observe the direction of the needle, and verify the following rule:

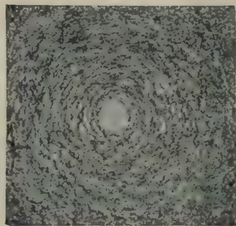


FIG. 139.—The magnetic field round a straight current.

Maxwell's corkscrew rule.—Imagine a corkscrew being screwed along the wire in the direction in which the current is passing. The direction in which the thumb rotates indicates the positive direction of the lines of force.

It should be borne in mind that, especially when the current is weak, the deflection of the compass-needle in some positions is much modified by the action of the earth's field.

EXPT. 153.—**Magnetic field due to a circular current.** Fig. 140 represents a grooved disc of wood, about 8 in. diameter, in the groove of which about 100 turns of cotton-covered copper wire are wound. The disc is fixed to the side of a small drawing-board so that the surface of the board coincides with a diameter of the disc.

Stretch a piece of thin white paper on the board, and support the board on a firm well-made box so that the

board can be turned round readily when desired. Connect the coil to a circuit including an accumulator, a rheostat and, if available, an ammeter. During the

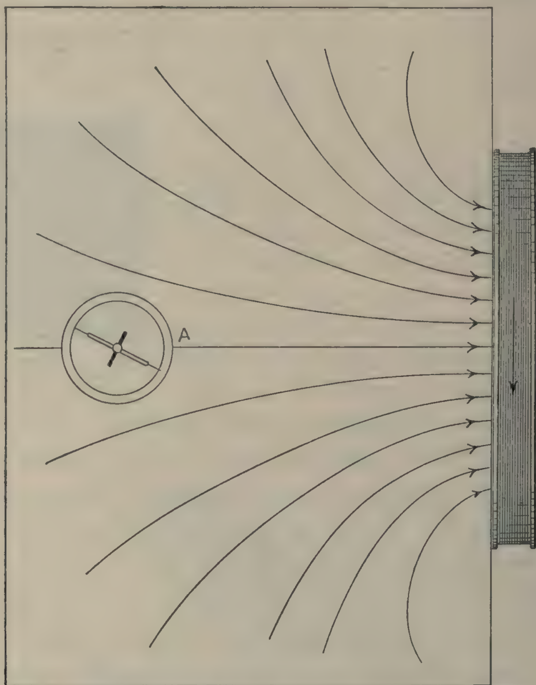


FIG. 140.—Magnetic field near a circular coil conveying a current.

experiment the current should be kept fairly constant; and this is ensured by means of the ammeter and rheostat.

(i) Plot the magnetic field by means of a sensitive compass-needle, always adjusting the position of the board so that the needle points in the direction of the earth's magnetic meridian. Mark the direction of

the magnetic force along the plotted lines, also the direction of the current round the coil.

Verify that when the near face of the coil, as seen by looking along the board, exhibits south-seeking polarity the apparent direction of the current is *clockwise*. Reverse the direction of the current, and notice a reversal of the direction of the magnetic field.

(ii) In order to observe the effect of increasing the strength of the current, adjust the board so that the plane of the coil coincides with the magnetic meridian. Connect the ends of the coil to a commutator, and include in the circuit as before an accumulator, a rheostat and an ammeter. Place at some point A, on the axis of the coil produced, a wide compass box containing a compass-needle fitted with a long pointer and circular scale. Notice that a current round the coil causes a deflection of the needle.

Adjust the current so that a deflection of about 10° is obtained; read accurately both ends of the pointer; reverse the current, and again read both ends of the pointer; read also the strength of the current as shown by the ammeter. Increase the current so that the deflection is increased by 7° or 8°, and repeat all the above readings. Repeat this for stronger currents until the deflection is about 60°. Tabulate your readings thus:

Strength of current (I).	Readings of pointer.	Mean deflection (θ).	tan θ.
1.	(i) ...... (ii) ...... (iii) ...... (iv)	......	......

Plot on squared paper the mean deflections (θ) and the current strength (I), taking the latter as abscissae (Fig. 141). If the curve is not a straight line, evidently *the current is not proportional to the deflection.* In the last column of the above table enter the **tangents** of the angles of deflection, and plot on the same squared paper the values of tan θ and of I. If the curve obtained is a straight line it

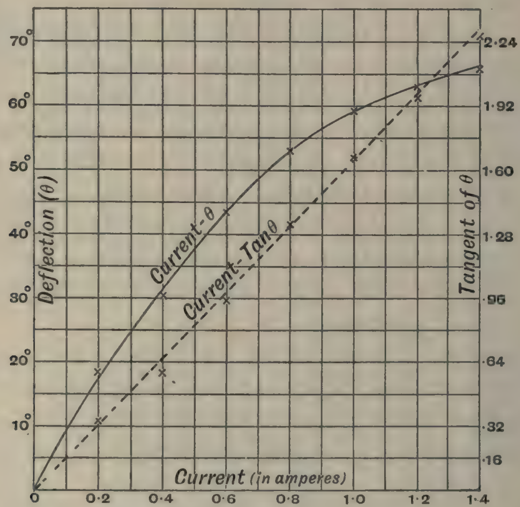


FIG. 141.—Observations taken with the apparatus of Fig. 140.

demonstrates that **the current is proportional to the tangent of the angle of deflection.** This is the principle of the **tangent galvanometer,** which supplies the definition of the unit in terms of which all currents are measured.

Ohm's law.—Ohm's law states that, in any simple conducting circuit which is kept at a uniform temperature, **the current** (I) is **proportional to the difference of electric potential** (P.D.) **between the ends of the conductor**; in other words, that the ratio P.D./I is a constant. Applying this statement to any complete circuit including some *source* of potential difference—such as a voltaic cell or a dynamo—it is customary

to use the expression **electromotive force** (E.M.F.) instead of "potential difference." If the whole circuit is considered, Ohm's law states that the ratio E/I is a constant, where E represents the electromotive force; whereas, if only a part of the circuit outside the cell or dynamo is considered, the law states that the ratio P.D./I is a constant. This constant is termed the **resistance** (R) of the part of the circuit under consideration. When E (or P.D.) and I are expressed in *volts* and *amperes* respectively, their ratio gives the resistance in *ohms*.

The truth of Ohm's law may be demonstrated by various methods; in the following experiment a portion only of a circuit is used, and the simultaneous values of P.D. and of I are observed by means of a voltmeter and ammeter.

EXPT. 154.—**Demonstration of Ohm's law.** In Fig. 142 AB is a long uncovered wire, preferably of an alloy

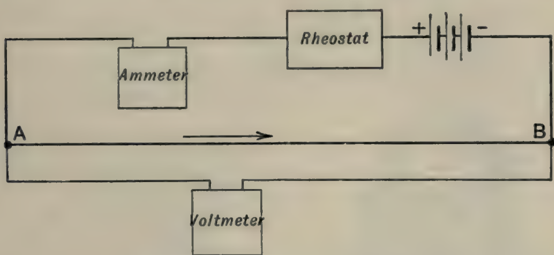


FIG. 142.—Demonstration of Ohm's Law.

such as manganin, the resistance of which is not affected appreciably by change of temperature. AB is part of a circuit which includes an ammeter, a rheostat and two or three accumulators. The terminals of a voltmeter are connected to the points A and B.

If a low-reading ammeter, with finely divided scale, is not available, a tangent galvanometer and commutator may be made to serve as an efficient substitute.

Instructions for using a tangent galvanometer are given on p. 217.

Adjust the rheostat so that the circuit is traversed by the smallest current which can be read accurately on the ammeter. Note the readings of both ammeter and voltmeter. Increase the current and repeat both readings. Continue this process until several independent pairs of readings are obtained. Tabulate your observations thus :

Potential difference (P.D.).	Current (I).	$\frac{\text{P.D.}}{\text{I.}}$

The Tangent Galvanometer.

The tangent galvanometer and its adjustment.—In any tangent galvanometer (Fig. 143) the relation between the current (I) and the deflection (θ) is expressed by the equation

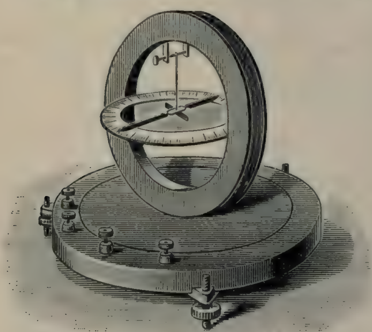


Fig. 143.—A tangent galvanometer.

$$I = \frac{Hr}{2\pi n} . \tan\theta \text{ absolute units,}$$

where H is the horizontal intensity of the earth's field, r the mean radius of the coil, and n the number of turns of wire in the coil. Since the ampere is equal to 0·1 absolute unit of current, the above equation may be written

$$I = \frac{10Hr}{2\pi n} . \tan\theta \text{ amperes.}$$

Providing that H, r and n are known, the current strength can be calculated for any observed deflection. But as the values of r and n for the instrument provided are frequently not known accurately, it is more satisfactory to determine the value of the factor $10\mathrm{H}r/2\pi n$ by an independent method—*e.g.* the electrolysis of copper (see p. 220). The above equation is then written

$$\mathrm{I} = k \tan \theta,$$

where k is the "reduction factor" of the instrument.

A tangent galvanometer is rendered more serviceable if wound with two or more separate coils, the coils being connected to different pairs of terminals; thus, when fitted with three coils and four terminals, it is possible to use either one coil only, or any pair of coils, or all three coils in series. A suitable design consists of (i) a coil of three or four turns of thick wire, (ii) a coil of 30 turns of thinner wire, and (iii) a coil of 60 turns of very thin wire.

Adjustments.—When using a tangent galvanometer, the following procedure must always be adopted:

(i) With a spirit-level, level the base of the instrument.

(ii) Place the instrument so that its coil is in the magnetic meridian.

(iii) The point of support of the needle may not be in the same vertical line as the centre of the circular scale. The error due to this is eliminated by reading both ends of the pointer.

(iv) The plane of the coil and the axis of the magnet may not be exactly in the magnetic meridian. Any error due to this is eliminated by reversing the direction of the current, and again reading both ends of the pointer.

Hence, whenever a tangent galvanometer is used either for measuring a current or for comparing two different currents, four readings of the pointer must be taken, as described above; and the mean of the four readings is taken as the deflection.

(v) The reading of the pointer should be taken always by viewing it with one eye only, situated vertically over the pointer, thus avoiding any error due to parallax. To assist in placing the eye correctly, the circular scale is usually mounted on a plane mirror.

EXPT. 155.—**Measurement of resistance (tangent galvanometer method).** Fig. 144 represents a circuit which includes an accumulator, a tangent galvanometer (T.G.), a resistance box and the unknown resistance (X). Use a galvanometer with a large number of turns of wire in its coil, and insert resistance (R_1) in the resistance box so that a deflection of

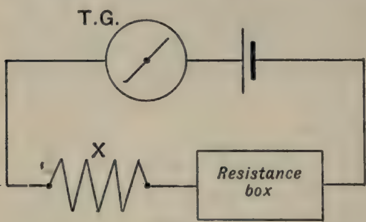


FIG. 144.

40°–50° is obtained. Assuming that the resistances of the galvanometer and of the cell are negligibly small compared with that of ($R_1 + X$), then, by Ohm's law,

$$I = E/(R_1 + X). \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Note the deflection. Remove X from the circuit, and increase the resistance of the resistance box to R_2, such that the same deflection as before is obtained; then

$$I = E/R_2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Hence, $R_1 + X = R_2$, or $X = R_2 - R_1$.

When two cells (E.M.F. denoted by E_1 and E_2) are connected in series with a tangent galvanometer, then tan α is proportional to ($E_1 + E_2$). If E_2 is then reversed, *tan* α will be less than before, and will be proportional to $E_1 - E_2$ (assuming that E_1 is greater than E_2). Hence

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{E_1 + E_2}{E_1 - E_2}, \quad \text{or} \quad \frac{E_1}{E_2} = \frac{\tan \alpha_1 + \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}.$$

EXPT. 156.—**Comparison of the E.M.F. of two cells (method of sum and difference).** This method can be carried out with a tangent galvanometer only when the cells differ considerably in their E.M.F. Connect the two cells *in series* with an adjustable resistance and

a tangent galvanometer; use a commutator with the galvanometer. Adjust the resistance so that the deflection is about 60°. Note the deflection of both ends of the pointer, and also when the commutator is reversed. Reverse the cell having the lower E.M.F., and repeat the observations. Record the observations thus:

Cells.	Deflections.		Mean deflection (α).	tan α.
	East End.	West End.		
In conjunction (E_1+E_2)	(i)	......	$\alpha_1=$......	tan $\alpha_1=$...
	(ii)	......		
In opposition (E_1-E_2)	(i)	......	$\alpha_2=$......	tan $\alpha_2=$...
	(ii)	......		

Calculate the value of the ratio E_1/E_2.

EXPT. 157.—**Grouping of cells.** Connect up in series one Leclanché cell, a resistance of 100 ohms, the commutator, and a tangent galvanometer. Note the deflection. Repeat with two cells in series, with three cells in series, and with three cells in parallel.

Repeat these observations, but use a 5-ohm resistance instead of the 100-ohm resistance. Enter your observations in the following manner:

Leclanché Cells.	Resistance of Coil.	Deflection.		Mean Deflection (α).	tan α.
		East End.	West End.		
1 cell	... ohms	......	......	......	......
2 cells in series	”	......	......	......	......
3 cells in series	”	......	......	......	......
3 cells in parallel	”	......	......	......	......

Note which arrangement of cells gives a maximum current through the low resistance, and through the high resistance.

Electro-chemical equivalents.—The electro-chemical equivalent of an element is the weight of it liberated, from a compound containing it, by the passage of one **coulomb** of electricity, *i.e.* by the quantity conveyed by a steady current of one ampere flowing for one second. The electro-chemical equivalents of silver and of copper have been determined with such extreme accuracy that these data are used frequently as a means of measuring a uniform current; the method serves also for determining the "reduction factors" of galvanometers and for verifying the accuracy of the readings of ammeters. The electro-chemical equivalent of silver is 0·001118 gm. per coulomb, and that of copper is 0·0003293 gm. per coulomb.

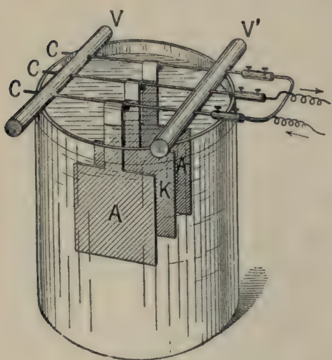


FIG. 145.—A copper voltameter.

The voltameter shown in Fig. 145 consists of two vulcanite rods (V and V′) joined together by three parallel thick wires of copper (C); this rests on the top of a beaker. The two outer copper plates (A), which serve as the *anode*, may conveniently be about 7 cm. × 5 cm.; the middle plate (K), which is the kathode, should be slightly smaller. As the electrolyte, use a 15% solution of copper sulphate to each litre of which 5 c.c. of strong sulphuric acid has been added.

EXPT. 158.—**The reduction factor of a tangent galvanometer (by deposition of copper).** Thoroughly clean the

copper plates with sand-paper. Connect up the battery, voltameter (V), rheostat (R), commutator (C) and tangent galvanometer (T.G.), as shown in Fig. 146, making sure that the anode plates are connected to the *positive* pole of the accumulator. Adjust R until a convenient deflection is obtained. Break the circuit, remove the kathode, wash it in water and dry it quickly *over* a spirit flame.

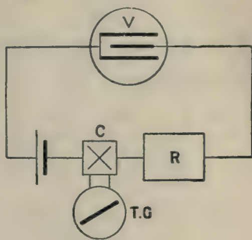


FIG. 146.

Weigh the kathode, and replace it in position. Note the time by your watch at the instant when the circuit is closed. Frequently observe the deflection and keep it constant by adjusting R. When 15–20 minutes have elapsed, *quickly* reverse the commutator and again note the deflection, keep it steady by the adjustment of R when necessary.

When the experiment has continued for not less than forty minutes, break the circuit and note the exact time when this was done. Remove the kathode, wash it in water, dry it over a spirit flame and again weigh it. Record the observations thus :

Time.	Weight of kathode.	Deflection.		Mean deflection (θ).
		East End.	West End.	
Initial, 11 h. 32′	13·6420 gm.	(i) $21^\circ\cdot5$ (ii) $23^\circ\cdot1$	(i) $23^\circ\cdot9$ (ii) $21^\circ\cdot1$	......$22^\circ\cdot4$
Final, 12 h. 32′	14·1254 ,,			

Weight of copper deposited (W) = 0·4834 gm.
Duration of experiment (t) = 3600 sec.

$$I = k \tan\theta,$$

or $$k = \frac{I}{\tan\theta} = \frac{0{\cdot}4834}{0{\cdot}0003293 \times 3600 \times \tan 22^\circ{\cdot}4} = 0{\cdot}993.$$

EXPT. 159.—**Calibration of an ammeter (by tangent galvanometer).** Connect up the apparatus exactly as shown in Fig. 146, except that the ammeter now takes the place of the copper voltameter (V). Adjust the rheostat (R) so that a deflection of about 10° is obtained in the tangent galvanometer. Note the reading of the ammeter, and the four readings of the pointer of the galvanometer. Increase the current slightly and repeat the readings. Continue this process until the deflection is about 70°. Tabulate the observations thus:

Constant (k) of the tangent galvanometer =

Ammeter reading.	Tangent galvanometer.		Current strength (I) $= k \tan\theta$.
	Pointer reading.	Mean deflection (θ).	
(i)	(i) ...... (ii) ...... (iii) ...... (iv)	......	......

Plot on squared paper the ammeter readings and the current strength, taking the former as abscissae.

RESISTANCE.

The Wheatstone bridge.—The principle of the Wheatstone bridge, for the measurement of a resistance, is shown in

Fig. 147. P and Q are two unknown resistances, but their ratio P/Q is known. R is a known resistance, and X is the resistance to be measured. When the ratio P/Q is adjusted so that there is no deflection of the galvanometer (G), then

$$R/X = P/Q,$$

or

$$X = R \times Q/P.$$

The student is recommended, when proceeding to make a measurement of resistance, always to sketch the above diagram and thereby verify the correctness of the several connections.

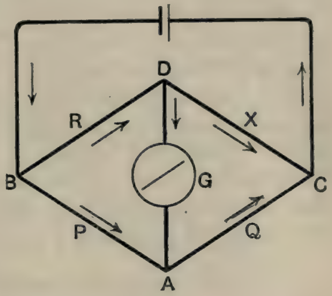


Fig. 147.—The Wheatstone bridge.

Fig. 148 represents a Wheatstone bridge (or "metre bridge," as it is sometimes termed) with the requisite connections completed. It consists of a uniform manganin wire, one metre long, stretched alongside a metre scale, and with

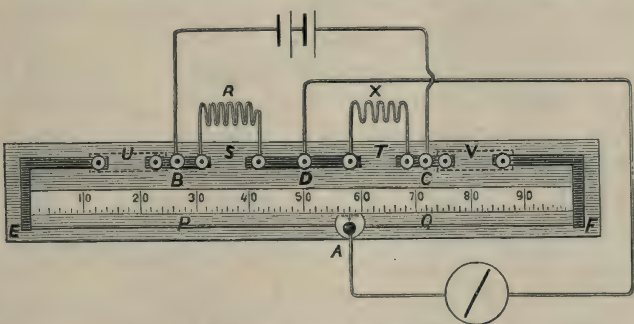


Fig. 148.—A metre bridge.

its ends soldered to stout copper strips E and F. There are usually four gaps between copper strips fixed along the other edge of the board; in simple measurements, however, the gaps U and V are closed by copper strips held in position by binding screws. The resistances R and X to

be compared are inserted in the gaps S and T. The battery is connected to the screws B and C; the galvanometer is connected to the screw D and the contact maker at A. The lettering in this illustration corresponds to that of the preceding diagram (Fig. 147), and the student should compare them carefully.

The galvanometer selected should be one in which the suspended needle rapidly comes to rest, otherwise there is much avoidable waste of time. Galvanometers of the tangent or astatic type are not suitable; a sensitive portable galvanoscope, such as that shown in Fig. 135 (p. 205), meets the requirements admirably.

EXPT. 160.—**The resistance of the coils of a tangent galvanometer.** Connect, *by short pieces of thick copper wire*, the terminals of the coil to the binding screws on either side of the gap T (Fig. 148). Similarly, connect a standard coil of known resistance to the binding screws of gap S. If several standard coils of different resistance are available, one which has a resistance approximating to that of X should be chosen.

When all connections are completed, move the contact maker A to a position near to E, *e.g.* to reading 5, make contact for a moment and note the direction in which the needle is deflected; move A to a position near to F, *e.g.* to reading 95, make contact for a moment and note whether the deflection is in the opposite direction. If so, the position of A for no deflection must be somewhere between 5 and 95. Repeat these observations at 10 and 90, and so on, until the point of contact for no deflection is found.

It is well to remember that *if the deflection is to the left (or right) the scale reading of A is too low, and if the deflection is to the right (or left) the scale reading is too high.* When the correct position of A has been found,

note the *lengths* of wire P and Q ; as the wire is quite uniform in material and diameter, the ratio of the resistances of P and Q will be the same as the ratio of their lengths.

Calculate the resistance of X from the equation

$$R/X = P/Q.$$

It is assumed that the wire is quite uniform, and that the junctions of the ends of the wire at E and F have no appreciable resistance. If these require verification, the student must apply tests which are described in more advanced text-books.

EXPT. 161.—**The resistance of two wires joined in parallel.** Obtain pieces of German-silver wire (or manganin) of different diameter ; Nos. 28 and 32 S.W.G. are suitable. Bare the ends of both wires and bend the bared ends to a right angle. Measure the resistance of each wire separately, taking care that the wire leaves the binding screws of the bridge just where the bend is situated. Insert the two wires, of which the resistances r_1 and r_2 are now known, in parallel between the binding screws, and measure the resistance. Enter the observations thus :

r_1.	r_2.	Resistance in parallel.	
		$\frac{r_1 r_2}{r_1 + r_2}$ (calculated).	By experiment.

The **specific resistance** of a metal is the resistance between opposite faces of a 1-cm. cube of the metal ; and since the resistance of any conductor is directly proportional

to its length and inversely proportional to its cross-section, the resistance may be expressed thus:

$$\text{resistance (R)} = \text{specific resistance } (k) \times \frac{\text{length } (l)}{\text{cross-section } (s)},$$

or
$$k = \frac{Rs}{l}.$$

EXPT. 162.—**The specific resistance of a metal.** Measure the length, diameter and resistance of a piece of manganin wire. Enter the results thus:

Metal.	Length (l).	Cross-section (πr^2).	Resistance (R).	$\frac{R \times \pi r^2}{l}$.
Manganin				

EXPT. 163.—**The resistance of a wire in relation to temperature.** Fig. 149 represents a spiral of iron wire (No. 28), about 2 metres long, with its ends soldered to short pieces of thick copper wire which pass through a cork fitted in a glass boiling tube. The apparatus is fitted with a thermometer and stirrer, and the tube is nearly filled with paraffin oil.

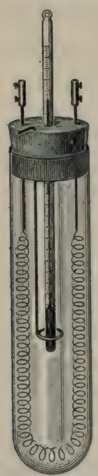


FIG. 149.—To illustrate Expt. 163.

Place a deep beaker full of water on a tripod, and fix the tube containing the wire spiral in the water. Connect up the ends of the spiral by means of thick copper wires to the binding screws of the bridge. After the tube has been in the water for about five minutes, stir the paraffin oil, and note the temperature. Measure the resistance of the spiral. Slowly warm the water, and frequently stir the oil. When the temperature has risen

about 10° C., remove the flame, stir the oil, and repeat the observations of temperature and resistance.

Repeat these readings at higher temperatures. From the first and the last observations determine how much a wire of 100-ohms resistance would increase in resistance due to a change in temperature of 1° C., thus,

$$\frac{R_2 - R_1}{R_1} \times \frac{100}{(T_2 - T_1)}.$$

Enter your observations thus:

Temperature.	Resistance.	Per cent. increase due to rise of 1° C.

The Post-office box.—A resistance can be measured more accurately by the Post-office box than by the metre bridge; and, if time permits, the student should make at least one measurement of a resistance by means of the former apparatus. In order to obtain the accuracy of result which the method is capable of affording, it is necessary to use a sensitive mirror galvanometer instead of a portable galvanoscope: a galvanometer of the "suspended coil" pattern, *e.g.* a d'Arsonval, is very suitable.

The Post-office box (Fig. 150), which is simply a compact form of Wheatstone bridge, consists of a number of resistance coils arranged so as to form the three resistance arms P, Q, and R of Fig. 147. (Corresponding points in Figs. 150 and 147 are denoted by the same letters.) Each of the arms BA and BD, known as "ratio arms," consist of three coils the resistances of which are 10, 100, and 1000 ohms respectively. The coils between *a′* and C constitute the third arm of the bridge, and their resistances are arranged so that any whole number of ohms, from 1 up to 10,000, is available.

The point A is permanently connected to a' by a thick wire passing under the cover of the box; another wire connects A to the stud of the key *a*, to which one terminal of the galvanometer is connected. The point B is similarly connected to the stud of the battery key *b*. These permanent connecting wires are indicated by the white lines drawn on the cover of the box. The battery terminals are connected to *b* and C; and the galvanometer to *a* and D.

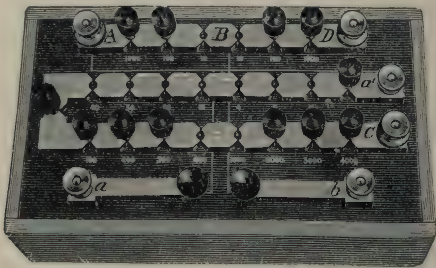


FIG. 150.—A Post-office box.

The resistance X to be measured is joined to D and C. In making the measurement, equal resistances are inserted in the ratio arms, and the resistance Q is adjusted until a minimum deflection of the galvanometer is observed; this gives the resistance of X to the nearest whole number. The ratio P/R is now made equal to 10, and the resistance of Q again adjusted as nearly as possible; Q is now approximately 10 times as great as X. Finally the ratio P/R is increased to 100, and Q again adjusted; X is then equal to Q/100.

THE POTENTIOMETER.

In Fig. 151 AB is a long uniform wire of small diameter, preferably of manganin, with a millimetre scale attached. The end A is connected to two or three constant cells (preferably accumulators), a tangent galvanometer (G_1), and a rheostat R. The end A must be joined to the +ve

pole of the battery. E_2 represents one of the cells to be compared, and with its +ve pole also connected to the end A. G_2 is a sensitive galvanometer, and R_2 is a

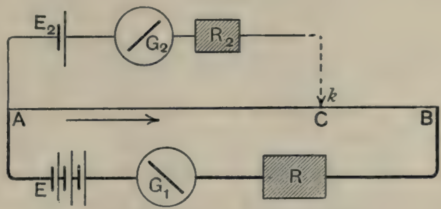


FIG. 151.—Principle of the potentiometer.

high resistance (5000–10,000 ohms). A movable knife-edge at k serves to make contact at any point of the wire AB. When the point of contact is adjusted so that there is no deflection in G_2, the E.M.F. of the cell E_2 is equal to the potential difference between the points A and C due to the battery E; and the length AC is proportional therefore to the E.M.F. of E_2.

EXPT. 164.—**Comparison of E.M.F. of voltaic cells (by the potentiometer).** During the experiment keep the deflection of G_1 constant by adjustment of the rheostat. Having made the necessary connections, make momentary contact between k and a point on AB near to the end A, and notice the direction of the deflection in G_2; then make contact at a point near to the end B, when the deflection should be in the opposite direction. If this is not so, either the connections are wrong, or the E.M.F. of the battery E is insufficient. If everything is working correctly, the point of contact for no deflection in G_2 is found in the same manner as in using the metre bridge.

Having noted the correct length AC, replace the cell at E_2 with the other which is to be compared, and repeat

the process. If available, take a reading with a standard cell (*e.g.* a Weston normal cell; E.M.F., 1·0184 volt at 20 °C.), and use this as a means of calculating the E.M.F. of the other cells tested. *Always keep the high resistance* R_2 *in series with* E_2 in order to prevent any appreciable current passing through the cell which might cause polarisation; but, *when the correct point of contact is apparently obtained*, R_2 may be considerably reduced or even removed entirely in order to increase the sensibility of the galvanometer and so to locate more accurately the correct point of contact.

EXPT. 165.—**Internal resistance of a cell (potentiometer method).** Use for the purpose a Leclanché cell, a dry cell, or a Daniell cell. Connect up the apparatus exactly as shown in Fig. 151, and find the length AC on the potentiometer corresponding to the E.M.F. of the cell. Without altering the connections, short circuit the cell through a 10-ohm coil, and find the new point of contact giving no deflection in G_2; this length represents the potential difference between the ends of the 10-ohm coil. As the cell may be affected by polarisation, the 10-ohm coil should be removed and the first length AC verified; if the two measurements of AC differ, the average value should be used in the calculation.

If R and r are the resistances of the coil and of the cell respectively, and if E and e are the lengths of the potentiometer wire required to balance (i) the cell on open circuit and (ii) the potential difference between the ends of the coil used, then

$$\left.\begin{aligned} E &= C(R+r) \\ e &= CR \end{aligned}\right\}.$$

Hence $$\frac{R+r}{R}=\frac{E}{e},$$

or $$r=R\times\frac{E-e}{e}.$$

Substitute the observed values of R, E and e, and calculate the values of r.

Heating Effect of Current.

Joule's law.—In any part of a simple electric circuit in which no mechanical or chemical work is done, the work done by the source of current in transmitting electricity through that part of the circuit reappears to an equivalent amount in the form of *heat.* Joule's law states that the heat generated in such a case is proportional (i) **to the square of the current strength,** (ii) **to the resistance**, **and** (iii) **to the time during which the current continues.**

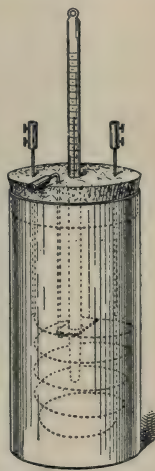


Fig. 152.—Apparatus for proving Joule's law.

Fig. 152 represents a form of apparatus suitable for the approximate verification of the law. The ends of a coil of thin manganin wire are connected to thick copper wires passing through a cork which fits a copper calorimeter. The coil is more convenient to manipulate if wound as a narrow spiral, bending round towards the bottom of the calorimeter, as shown in Fig. 93 (p. 142). The cork also carries a thermometer (with scale reading to 0°·2 C.) and a stirrer. Two of these appliances should be available and have different resistances (*e.g.* 4 ohms and 8 ohms).

Expt. 166.—**Verification of Joule's law.** (i) Measure accurately the resistances of the two coils to be used.

Connect them up in series with a constant cell, a tangent galvanometer and commutator (or, a low-reading ammeter), and a rheostat, as shown in Fig. 153. Adjust R so as to obtain a deflection of about 30°. Break the circuit.

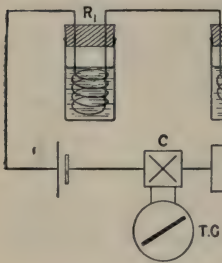


FIG. 153.

Dry and weigh the calorimeters, and pour equal weights of cold water into each of them, sufficient in quantity to cover the spirals. Wrap a layer of cotton-wool round each calorimeter to minimise loss of heat by radiation. Note the reading of each thermometer. Note the time at the instant when the circuit is completed. Frequently stir the water in each vessel, note the reading of both ends of the pointer, and keep the deflection constant by means of the rheostat. At the end of ten minutes note the temperatures, quickly reverse the commutator, note the deflection, and continue the experiment for exactly ten minutes longer.

Record the observations thus:

$$\left.\begin{array}{l}\text{Resistance of } R_1 = \quad \text{ohms.}\\ \text{Resistance of } R_2 = \quad \text{ohms.}\end{array}\right\}$$

Time.	Temperature of R_1.	Temperature of R_2.	Deflection.		Mean Deflection (α_1).	tan α_1.
			East End.	West End.		
10 h. 30 m.	......	......	....	....	}	....
10 h. 40 m.	......	......	....	....		
10 h. 50 m.	......	......				

$$\text{(A)} \quad \frac{\text{Rise in temperature of } R_1 \text{ in 10 minutes}}{\text{Rise in temperature of } R_1 \text{ in 20 minutes}} = \ldots\ldots$$

$$\frac{\text{10 minutes}}{\text{20 minutes}} = \ldots\ldots$$

The equality of these ratios proves that *the amount of heat generated by the current is proportional to the time.*

$$\text{(B)} \quad \frac{\text{Rise in temperature of } R_1 \text{ in 20 minutes}}{\text{Rise in temperature of } R_2 \text{ in 20 minutes}} = \ldots\ldots$$

$$\frac{\text{Resistance of } R_1}{\text{Resistance of } R_2} = \ldots\ldots$$

The equality of these ratios proves that *the amount of heat generated is proportional to the resistance.*

(ii) Remove R_1 from the circuit, and carry out the following experiment with R_2: Thoroughly dry the calorimeter and pour into it exactly the same weight of cold water as used in Expt. 166 (i). Adjust R so as to obtain a greater deflection than before. Insert the spiral into the calorimeter and proceed exactly as in Expt. 166 (i), taking special care to keep the deflection constant, and also reversing the current two or three times so as to obtain the true mean deflection. Enter your results thus:

Time.	Temperature of R_2.	Deflection.		Mean Deflection (α_2).	tan α_2.
		East End.	West End.		
11 h. 0 m.	....	....	....	....	....
11 h. 10 m.	....	....	....		
11 h. 20 m.	....				

Compare the rise in temperature in this experiment with that obtained in the same calorimeter in Expt. 166 (i), and calculate the following ratios :

$$\frac{\text{Rise in temperature in Expt. ii.}}{\text{Rise in temperature in Expt. i.}} = \ldots\ldots$$

$$\frac{(\tan \alpha_2)^2}{(\tan \alpha_1)^2} = \ldots\ldots$$

This result should verify the fact that the heat developed is proportional to the *square of the current.*

PHYSICAL TABLES.

(1) Imperial Standard Wire Gauge.

Descriptive Number.	Diameter.		Area of Cross-Section.	
	Inches.	Centimetres.	Square Inches.	Square Centimetres.
14	0·080	0·203	0·00503	0·0324
15	0·072	0·183	0·00407	0·0263
16	0·064	0·163	0·00322	0·0207
17	0·056	0·142	0·00246	0·0159
18	0·048	0·122	0·00181	0·0117
19	0·040	0·102	0·00126	0·00811
20	0·036	0·0914	0·00102	0·00657
21	0·032	0·0813	0·000804	0·00519
22	0·028	0·0711	0·000616	0·00397
23	0·024	0·0610	0·000452	0·00292
24	0·022	0·0559	0·000380	0·00245
25	0·020	0·0508	0·000314	0·00203
26	0·0180	0·0457	0·000254	0·00164
27	0·0164	0·0417	0·000211	0·00136
28	0·0148	0·0376	0·000173	0·00111
29	0·0136	0·0345	0·000145	0·00094
30	0·0124	0·0315	0·000121	0·00078
31	0·0116	0·0295	0·000106	0·000682
32	0·0108	0·0274	0·000092	0·000591
33	0·0100	0·0254	0·000079	0·000507
34	0·0092	0·0234	0·000066	0·000429
35	0·0084	0·0213	0·000055	0·000357
36	0·0076	0·0193	0·000045	0·000293

(2) Maximum Pressure of Aqueous Vapour.

(The pressure p is given in mm. of mercury, at 0° C.)

Temp. °C.	p.	Temp. °C.	p.	Temp. °C.	p.
0	4·57	21	18·47	96	657·4
1	4·91	22	19·63	97	681·9
2	5·27	23	20·86	98	707·1
3	5·66	24	22·15	98·2	712·3
4	6·07	25	23·52	98·4	717·4
5	6·51	26	24·96	98·6	722·6
6	6·97	27	26·47	98·8	727·9
7	7·47	28	28·07	99	733·2
8	7·99	29	29·74	99·2	738·5
9	8·55	30	31·51	99·4	743·8
10	9·14	40	54·87	99·6	749·2
11	9·77	50	91·98	99·8	754·7
12	10·43	60	148·9	100	760·0
13	11·14	70	233·3	100·2	765·2
14	11·88	80	354·9	100·4	771·0
15	12·67	90	525·5	100·6	776·5
16	13·51	91	545·8	100·8	782·1
17	14·40	92	566·7	101	787·7
18	15·33	93	588·3	102	816·0
19	16·32	94	610·6	103	845·3
20	17·36	95	633·7		

EXERCISES (NUMERICAL AND PRACTICAL).

CHAPTER I.

1. A spherometer was adjusted for true contact on the flat surface of a sheet of plate glass. The mean distance between the screw point and the legs was found to be 2·26 cm. The pitch of the screw was 0·5 millimetre, and the circumference of the horizontal disc was divided into 100 equal divisions. In order to make true contact when supported on a concave surface of a diverging lens, the screw was rotated through 286 divisions. Calculate the radius of curvature of the concave surface.

2. A circular scale of a spectrometer is divided into half-degrees; and, on the vernier scale attached to it, 29 of these divisions are divided into 30 equal parts. The scale-reading is between $83^\circ{\cdot}5$ and 84°, and the two scales coincide at the 22nd division of the vernier. What is the correct scale-reading?

3. The inch-scale of a standard barometer is divided into twentieths of an inch: and, on the vernier scale attached to it, 24 of these divisions are divided into 25 equal parts. The scale-reading is between 29·5 and 29·55, and the two scales coincide at the 15th division of the vernier. What is the correct scale-reading?

4. Find, by experiment, the length of a pendulum which completes one oscillation in two seconds.

CHAPTER II.

1. Given a scale divided into tenths of an inch, construct a vernier with the aid of which lengths may be measured

to one hundredth of an inch, and make measurements with it to find the area of the given surface.

2. Find, in two ways, the ratio of the areas of the surfaces of the two given sheets of tinfoil (of equal thickness).

3. On squared paper describe a circle of 3-inch radius, and describe a square within the circle. Determine the difference between the areas of the circle and the square. Verify the result by calculation from the dimensions of the diagram.

4. Describe five circles, each of 3-cm. radius, and describe within these circles regular polygons having 3, 4, 5, 6 and 8 sides respectively. Calculate, by geometry, the areas of the polygons. Plot these areas on squared paper, taking the number of sides as abscissae. From the curve estimate the area of the circle, and verify this result by calculation.

5. In a water reservoir the following figures give the area (in sq. ft.) of the water surface when filled to different heights (h) above the lowest point of the bottom:

Height, -	0	5	10	15	20	25	30	35	40
Area, -	0	5100	10400	16400	23100	31000	39800	50000	61200

Find, by Simpson's rule, the total volume of water when the reservoir is filled to a height of 40 ft.; also find the volume of water supplied by the reservoir when the height falls from 40 ft. to 20 ft.

CHAPTER III.

1. Find the volume of 50 lead shot by the specific gravity bottle.

2. You are provided with a burette, a millimetre scale, gummed paper and a test-tube. Determine the internal diameter of the test-tube.

3. Assuming that the given burette is graduated correctly, determine the corrections for each 10 c.c. mark on a graduated cylinder.

4. By means of a screw gauge and calipers determine the volume of the given piece of glass rod. Verify the result by means of a balance and a relative density bottle.

5. By means of a burette and a measuring cylinder graduated in cubic inches, determine the number of c.c. equivalent to one cubic inch.

CHAPTER IV.

1. In weighing an object by the *method of vibrations*, the following readings were obtained. Calculate the true weight of the object to the nearest tenth of one milligram.

Object: A recently coined silver shilling.

Readings:

(i) With no load.		(ii) With 5·657 gm. on right pan.		(iii) With 5·658 gm. on right pan.	
L.	R.	L.	R.	L.	R.
7·6		9·8			12·9
	12·0		14·0	4·8	
7·8		9·9			12·5
	11·8		13·9	5·2	
8·0		10·0			12·0

On the assumption that five shillings worth of silver coins should weigh exactly one ounce, calculate the percentage error in the weight of the coin.

2. In a measurement of the diameter of a piece of thermometer-tubing, with circular bore, by the method of weighing a mercury thread of observed length, the following data were obtained:

Length of mercury thread, - -	21·9 cm.
Temperature, - - - -	14° C.
Weight of crucible, - - -	18·546 gm.
" " " + mercury, -	24·942 "

If the density of mercury at 14° C. is 13·56 gm. per c.c., calculate the diameter of the tube.

3. Find the sensibility of the given balance, (i) with no load, and (ii) with 50 gm. on each pan.

4. The balance provided is hopelessly out of adjustment. Find, by means of it, the weight of the solid provided.

5. You are provided with a spring balance (with an arbitrary scale), two weights of 10 gm. and 50 gm. respectively, an irregular solid and some squared paper. Find the weight of the irregular solid.

6. Determine the errors in the scale of the given burette, and plot the corrections on squared paper.

7. You are provided with a balance, a box of weights, a small glass flask with a horizontal file-mark on the neck, and some cold water. Find the capacity of the flask up to the file-mark. Find also the volume of water delivered when the contents are poured out.

8. A balance, a box of weights, a 1-oz. weight, some sheet lead and a pair of strong scissors are provided. Make from the lead two weights each equal to 0·1 oz. Finally determine the difference (if any) between them. Record all details of your weighings.

9. Find the average internal diameter of the given capillary tube by means of a mercury thread.

10. Measure the diameter and the length of a metal cylinder with vernier calipers, calculate the volume of the cylinder, weigh it, and determine its density from your observations.

CHAPTER V.

1. If the height of the barometer is 76 cm. of mercury, calculate the pressure of the atmosphere (*a*) in lb. per sq. in., (*b*) in gm. per sq. cm. [Density of mercury $= 13{\cdot}55$ gm. per c.c.; 1 lb. $= 453{\cdot}6$ gm.]

2. A barometer, which originally records a height of 75 cm., is lowered into water until the surface of the water

and the upper level of the mercury coincide. Given that the specific gravity of mercury is 13·6, calculate the new height of the mercury column.

3. The top of a barometer tube of 1 sq. cm. sectional area is 80 cm. above the level of the mercury in the cistern, and the height of the mercury column is 74 cm. above the same level when there is no air within the tube. On introducing an air bubble up the tube, the level drops to 64 cm. What volume did the air that has been introduced occupy before its introduction?

CHAPTER VI.

1. Find the relative density of sand, compared with water at the temperature of the experiment, from the following data:

Mass of specific gravity bottle, empty,	23·32 gm.
Mass of specific gravity bottle, partly filled with sand, - - - -	81·69 „
Mass of specific gravity bottle containing the sand and filled up with water, - - - - - -	109·66 „
Mass of specific gravity bottle filled with water alone, - - - -	73·53 „

2. A certain cylinder, the area of its circular ends being 100 sq. cm., and its length 5 cm., floats in salt water of specific gravity 1·03 with its axis vertical. The weight of the cylinder is 400 grams. What fraction of its length is above the surface of the liquid, and what force must be applied to the cylinder to bring its upper surface into the level of the liquid?

3. A body, weighing 25 grams in air, weighs 21 grams when immersed in water and 23 grams when immersed in oil. What is the relative density of the oil?

4. A beaker containing water rests on one pan of a balance, and is exactly counterpoised by weights in the

other pan. A mass of metal, weighing 1 lb., is supported in the hand by means of a string, and is let down into the water but without touching the beaker. It is found that 2 ounces have to be added to the other pan to restore equilibrium. Find the specific gravity of the metal.

5. (i) Calculate the relative densities of glass rod and of dilute brine from the following data obtained with a Nicholson hydrometer:

Weight of hydrometer, - - -	95·04 gm.
Weight added to upper pan to sink it in cold water to the mark, - -	2·76 „
With the glass rod on the upper pan, the additional weight required on upper pan, - - - - -	1·10 „
With the glass rod on lower pan, the additional weight required on upper pan, - - - - - -	1·76 „
With hydrometer alone, floating in brine, the additional weight required on upper pan, - - - -	10·64 „

(ii) The relative density of the same brine was measured by a Twaddle hydrometer. The scale reading was 16·1. Use this figure in order to verify the accuracy of the result obtained with the Nicholson hydrometer.

6. A pencil has a piece of metal attached to one end. It floats in water with 3 cm. projecting above the surface and in alcohol with 1 cm. projecting. The total volume of metal and pencil is 5 c.c. and the cross-section of the latter is 0·45 sq. cm. Find the relative density of the alcohol.

7. Pour sufficient lead shot into a test-tube to enable it to float vertically in water and in spirit. Sink it to the same depth in each by appropriate adjustment of the shot, and determine from suitable weighings of the tube and shot the relative density of the spirit.

8. Determine the external diameter of a test-tube, from observations of the depth to which the test-tube sinks in water when different weights are placed within it.

9. Find the relative density of a given liquid by weighing a metal in it, in air and in water.

10. Determine the relative density of powdered sugar, and explain in detail how the result is deduced from the observations.

11. Find the density of the given solid by the specific gravity bottle.

12. Find by two different methods the specific gravity of the oil supplied.

13. You are provided with a balance, a box of weights, a bridge, a beaker, cotton thread, a metre scale and a reel of fine uncovered metal wire. Find by experiment the diameter of the metal wire.

14. Weigh accurately 20 gm. of dry common salt and dissolve it in water sufficient to make 100 gm. of solution. Find its relative density at 15° C. by the most accurate method available.

15. By means of a balance, weights, beaker and thread, find the volume of the irregular solid.

16. You are provided with a balance, weights, cotton thread, a beaker, an irregular lump of metal, some water and some milk. Find the relative density of the milk.

17. By means of a specific gravity bottle, find the relative density of the powdered glass provided.

18. You are provided with a specific gravity bottle, common salt, water, petroleum oil, a balance and weights. Thoroughly dry the salt and determine its relative density.

19. You are provided with a balance, weights, beaker, glass stopper, cold water, turpentine and a large crystal of alum. Without breaking the crystal, find its relative density.

20. Find how the amount of extension of the given spiral spring depends upon the stretching force, and use the spring to determine the specific gravity of the given body.

CHAPTER VII.

1. A flexible string 17 inches long has its ends attached to a horizontal beam at two points 13 inches apart. A mass of 2 lb. is suspended from the string at a point 5 inches from one end. Determine the tensions in the two portions into which the string is thus divided.

2. A uniform plank 20 ft. long is supported at one end and at a point 9 ft. from the other. A man weighing 10 stone walks along the plank until, when he is 2 ft. from the unsupported end, it overturns. Find the weight of the plank, and also the pressures on its supports when the man is at its centre.

3. Uniform rods AB, BC and CD, of equal lengths, rigidly connected at B and C, and weighing 1, 2 and 3 lb. respectively, form three sides of a square. Find the centre of gravity of the system.

4. A small disc, 0·8 inch radius, is cut out of a larger disc, 2 inch radius, of uniform cardboard. If the centre of the small disc is 1·0 inch distant from the centre of the larger disc, calculate the distance from this centre of the 'centre of gravity' of the remainder.

5. A circular hole of radius a is cut in a circular disc of wood of radius b, the distances between the centres of the hole and disc being c. The hole is then filled with a metal disc of the same thickness as the wood and of ρ times its density. Find the distance of the centre of mass of the loaded disc from the centre of the wood disc.

6. Prepare a 25 cm. square piece of uniform cardboard, and cut from it an equilateral triangle whose side is equal to and coincides with one of the sides of the square. Find

by experiment the position of the centre of gravity of the remaining portion, and verify the result by calculation.

7. Find the local value of g, by means of a simple pendulum.

8. Prove experimentally that when three forces are in equilibrium, each force is proportional to the sine of the angle between the other two.

9. Suspend by a long string a weight of one kilogram. Attach to the weight a second string connected to a spring-balance, and by means of this apply a deflecting force sufficient to deflect the first string until it is inclined 30° to the vertical, while the second string is inclined upwards at 60° to the vertical. Read the spring-balance. Verify the result geometrically by drawing a 'force diagram' to scale.

10. To a point near one end of a uniform wooden rod, about 3 ft. long, attach a weight of 1 oz. Lay the rod on the surface of a table (with a rectangular edge) so that the weight hangs beyond the edge. Gradually slide the rod further off the table until it is just on the point of overbalancing. Take such measurements as will enable you to calculate the weight of the rod.

11. Arrange a uniform wooden rod, about 3 ft. long, so that it can move freely in a vertical plane with its lower end resting on a table. Suspend from its middle point a weight of one kilogram; and attach a spring-balance, by means of string, to a point near to the upper end of the rod. Keeping this string always at right angles to the rod, increase the tension until the rod is inclined at 60° to the horizontal. Read the tension in the spring-balance. Verify the result theoretically by taking moments round the lower end of the rod.

12. A long smooth wooden board and a rectangular wooden block are supplied. Gradually tilt the board until the block, when once started, moves *with uniform velocity* down the board. The coefficient of friction is equal to the tangent of the angle of inclination of the board. Take

the measurements necessary to calculate the coefficient. Repeat the observation with different weights attached to the upper surface of the block.

CHAPTER VIII.

1. A cast-iron pulley wheel, consisting of one sheave weighing 1·015 lb., was suspended from a beam. A load of known weight was attached to one end of a cord passing over the wheel, and a scale-pan of known weight was attached to the other end. Weights were added to the pan until this total weight (or *Force*) was sufficient to raise the load at constant speed. The following data were obtained:

Load (**W**).	Force (**F**).	Friction (**F** − **W**).	Ratio, $\frac{\mathbf{F}-\mathbf{W}}{\mathbf{F}+\mathbf{W}+\text{Sheave}}$.
2 lb.	2·235 lb.	....	....
4 "	4·375 "	....	....
6 "	6·58 "	....	....
8 "	8·76 "	....	....
12 "	13·07 "	....	....
16 "	17·34 "	....	....
20 "	21·69 "	....	....
24 "	26·00 "	....	....

Fill in the remaining figures, and plot on squared paper the friction (**F** − **W**) and the load (**W**), taking the latter as abscissae. State whether the curve suggests any simple relationship between the friction and the load. The total load on the bearings should include the weight of the sheave. Is the ratio in the fourth column constant?

2. The following observations were obtained with a two-sheaved pulley (Fig. 62). Calculate for each reading the *mechanical advantage* (**W**/**F**) and the *efficiency*

$$(\mathbf{W}/(\mathbf{F}\times\text{velocity-ratio})).$$

The velocity-ratio was 4. Plot on squared paper the values

of **W** and **F**, taking the loads as abscissae, and on the same paper plot the efficiency-load curve.

Load (**W**).	Force (**F**).
4 lb.	1·112 lb.
5 „	1·372 „
6 „	1·628 „
7 „	1·894 „
8 „	2·153 „

3. Find (i) the mechanical advantage and (ii) the efficiency of the given system of pulleys, using at least six different loads. Plot your results on squared paper.

4. Describe how you would obtain a *mechanical advantage* of 4 by means of (1) an inclined plane, and (2) a system of pulleys, if there were no friction. Show how each case illustrates the statement that "what is gained in power is lost in speed."

CHAPTER IX.

1. The coefficient of expansion of water was determined by the following method: The cross-section of some narrow glass tubing (about 2 mm. bore) was measured by weighing a mercury thread of observed length. A bulb was blown on the end of the tube, and two file-marks (about 5 cm. apart) were made on the tube above the bulb. The tube was weighed, and boiled distilled water was introduced into the tube, and in such quantity that when cooled its surface was just below the lower file-mark. It was then immersed in a water bath, which was slowly warmed; and the temperatures were noted when the water surface coincided with each of the file-marks. Calculate the mean apparent coefficient of expansion of water from the following data:

(i) *Cross-section of tube.*

Length of mercury column, - -	11·25 cm.
Temperature of room, - - -	17°·5 C.
Weight of empty crucible, - -	7·484 gm.
„ crucible + mercury, - -	13·724 „

[Density of mercury, at 17°·5 C., 13·55 gm. per c.c.]

(ii) *Expansion.*

Distance between file-marks, - -	5·01 cm.
Weight of empty glass bulb, - -	39·00 gm.
" bulb + water, - - -	53·57 "
Temperature, when surface coincides with lower mark, - - -	25°·2 C.
Temperature, when surface coincides with upper mark, - - -	57°·5 C.

Calculate the mean *apparent* coefficient of expansion between 25°·2 C. and 57°·5 C. Also, assuming that the coefficient of cubical expansion of glass is 0·000025, calculate the mean *real* coefficient of expansion of water between the same temperatures.

2. In an experiment to determine the coefficient of expansion of glycerine, a sealed glass bulb containing mercury was suspended by a long fine wire from the pan of a balance and weighed (i) in air, (ii) in cold glycerine, and (iii) in warm glycerine, the temperatures of the glycerine being accurately observed, the following data were obtained:

Weight of bulb (in air), - - -	29·401 gm.
" " (in glycerine, at 13°·6 C.),	14·704 "
" " (" 65°·2 C.),	15·070 "

Calculate the apparent coefficient of glycerine; and prove that the *true* coefficient is obtained by adding to this result the cubical coefficient of expansion of glass (0·000025).

3. A vessel, 15 cm. long, 10 cm. broad, and 5 cm. deep at 0° C., is made of a metal of which the coefficient of *linear* expansion is 0·00002, and is filled with a liquid of which the coefficient of *cubical* expansion is 0·0002. What fraction of the liquid will escape if the vessel is heated to 20° C.?

4. A glass flask, when filled to a mark in the neck, contains 100 gm. of mercury at 0° C. Find how many grams of mercury it would contain at 100° C. if the mercury alone

were to expand, the coefficient of expansion of mercury being 0·00018.

5. A certain iron pendulum makes a complete swing from side to side in one second at 0° C. Being given that the time of swing of a pendulum is proportional to the square root of its length, and that the coefficient of linear expansion of iron is 0·000012, find how long the pendulum will take in executing one swing at 20° C.

6. A mass of gas, kept at constant volume, indicates a pressure of 75 cm. of mercury when immersed in a vessel containing melting ice, and a pressure of 54·5 cm. when the ice is replaced by a certain liquid. Calculate the temperature of this liquid.

7. Find the corrections to the fixed points of the thermometer supplied. [The boiling-point of water changes by 0°·037 C. for each 1 mm. change in the height of a mercury barometer.]

8. By means of a specific-gravity bottle, find the apparent coefficient of expansion of water (i) between 20° C. and 40° C., (ii) between 40° C. and 60° C., and (iii) between 60° C. and 80° C.

9. Find the temperature at which the given liquid becomes of the same density as water by heating it and water together in the same vessel and observing what happens as the temperature rises.

CHAPTER X.

1. Calculate the specific heat of copper from the following data, obtained by the method of mixtures:

Weight of copper calorimeter, - -	49·1 gm.
,, ,, ,, + water,	100·2 ,,
,, hot copper, - - - -	54 ,,
Temperature of hot copper, - -	97° C.
,, cold water, - - -	16°·2 ,,
Final temperature, - - - -	22°·9 ,,

2. In an experiment to determine the specific heat of turpentine, a known weight of hot iron was mixed with a known weight of the cold oil. The following data were obtained:

Weight of copper calorimeter,	- -	22·44 gm.
" " "	+ turpentine,	121·76 "
" cast-iron nails,	- - -	47·44 "
Temperature of hot water,	- - -	98°·4 C.
Initial temperature of oil,	- - -	17°·04 "
Final " "	- - -	25°·60 "

If the specific heats of iron and of copper are 0·112 and 0·095 respectively, calculate that of turpentine.

3. About 50 c.c. of ice-cold water was poured into a small calorimeter (previously cooled in ice). The outside was quickly dried and placed on gauze over a steady flame. It was found that the water began to boil in 2 min. 50 sec., and that at the end of a further period of 14 min. 40 sec. the water had all boiled away. Make a rough calculation of the latent heat of vaporisation of water.

4. Twelve grams of air are passed very slowly through a copper coil contained within a copper calorimeter filled with water. The combined weight of calorimeter and coil is 200 grams, and the weight of the water is 100 grams. The air enters the coil at a temperature of 100° C., while during its passage through the coil the temperature of the water rises from 19° C. to 21° C. Find the specific heat of air. [Specific heat of copper = 0·1.]

5. Heat 200 grams of water at as uniform a rate as possible over a Bunsen flame, and construct a graph showing the variation of temperature of the water with time until the water has boiled for 10 minutes. Comment upon any deviation of the graph from a straight line. Weigh the remaining water and calculate its latent heat of evaporation.

6. Determine the water equivalent of the given calorimeter by experiment.

7. Find the specific heat of a given metal by the method of mixtures.

8. Find the specific heat of the given dilute acid.

CHAPTER XI.

1. Dissolve 10 grams of common salt in 200 c.c. of water, and find the boiling point of the solution.

2. Given a bright metal vessel and some ice, find the dew point, and, with the aid of tables of vapour pressure, deduce the pressure of aqueous vapour in the atmosphere.

CHAPTER XII.

1. Calculate the value of Joule's equivalent from the following data obtained by the cardboard tube and lead shot method (p. 141):

Weight of lead shot, - - - -	443·5 gm.
Height through which the shot fall, -	68 cm.
Number of times the tube was inverted,	50
Initial temperature of shot, - - -	19°·05 C.
Final ,, ,, - - -	21°·60 ,,

[Specific heat of lead, 0·031.]

2. The following observations were obtained in a determination of the *mechanical equivalent of heat* by the electrical method:

Weight of copper calorimeter, - -	62·1 gm.
,, ,, calorimeter + cold water, -	267·3 ,,
Temperature of room, - - -	17°·5 C.
Initial temperature of water, - -	13°·2 ,,
Final ,, ,, - -	21°·8 ,,
Voltmeter reading, - - - -	9·0 volts.
Ammeter ,, - - - -	1·58 amperes.
Duration of experiment, - - -	9 minutes.

[Specific heat of copper, 0·095.]

Calculate the mechanical equivalent, expressed in ergs per calorie.

3. How much work must be expended in rubbing together two pieces of ice at the melting point to produce 10 gm. of water?

Latent heat of fusion of ice = 80 thermal units.

Mechanical equivalent of heat $= 4 \cdot 2 \times 10^7$ ergs per thermal unit.

CHAPTER XIII.

1. When using a grease-spot photometer, a balance is obtained when one of two lamps being compared is at a distance of one foot from the grease spot. The glass of this lamp, which was dirty, is cleaned, and the lamp has now to be placed three inches further away from the spot in order to obtain a balance. What fraction of the candle-power was being wasted before the lamp glass was cleaned?

2. By means of two sources of light and a Bunsen photometer, determine what fraction of the light falling upon it is absorbed by a thick sheet of plate glass.

3. Two plane mirrors are placed parallel to and facing one another at a distance apart of 12 inches, and a lighted candle is placed between them and 4 inches from one mirror. Make a diagram showing the paths of two rays by which an eye looking towards this mirror might see any one point of the flame by rays that had undergone three reflections, and show where the image formed by these rays would appear to be situated.

4. Place two strips of plane mirror vertically on a sheet of paper, and with their reflecting surfaces inclined at 60°. Fix a pin vertically at any point between the mirrors. Count the number of separate images which can be seen, and locate them by the parallax method. Assuming that the eye is placed in a given position, make a diagram showing the paths of the rays giving (i) an image due to *two* reflections, and (ii) an image due to *three* reflections.

5. Plot a curve showing the relationship between the thickness of a plate and the displacement of a ray incident on it at an angle of 25°.

6. Plot a curve showing the relationship between the angles of emergence and of incidence for a ray passing through a given prism.

7. Place a short piece of steel knitting-needle at the bottom of a tall glass cylinder full of water. Support at some distance above the water a small pointed gas-flame, using for the purpose a piece of drawn-out glass tubing. View the experiment from above, and adjust the position of the flame so that the position of its image seen by reflection from the water surface coincides with the apparent position of the needle. Measure the depth of the water and the vertical distance of the flame above the water surface; and from these measurements calculate the refractive index of water.

8. A ray of light falls upon a face of an equilateral glass prism, making an angle of incidence of 60°. Show how to trace the path of the ray through the prism and into the air beyond, assuming the index of refraction of light passing from air to the glass of the prism to be 1·6.

9. Find the index of refraction of the material of a given plate.

10. Find the angle of least deviation for a ray passing through a given prism, and obtain the values of the angles of incidence and emergence in this position.

CHAPTER XIV.

1. The following data were obtained in an experiment to find the focal length of a converging lens, by the method of illuminated cross-wires and real image (see p. 160).

	u	v
(i)	37·7 cm.	71·6 cm.
(ii)	41·75 "	60·7 "
(iii)	44·25 "	55·9 "

Calculate from the three independent sets of observations the mean focal length of the lens.

2. In order to find the focal length of a diverging lens it was fastened to a more powerful converging lens, the focal length of which was previously found to be $-14{\cdot}08$ cm., and the focal length of the combination was measured by the same method as in Exercise 1. The following data were obtained:

u	v
47·7 cm.	49·5 cm.

Calculate the focal length of the combination, and also that of the diverging lens.

3. In an experiment with a concave mirror the following measurements of u and v were obtained:

u	v	u	v
27 cm.	77 cm.	60 cm.	30 cm.
30 „	60 „	80 „	26·7 „
40 „	40 „	100 „	25 „
50 „	33 „	120 „	24 „

Plot the readings on squared paper, and find from the curve the positions of the image when (i) $u=35$ cm., (ii) $u=70$ cm., (iii) $u=90$ cm.

4. The distance between a source of light and a screen is 100 cm. In what positions can a convex lens of focal length 16 cm. be placed so as to form an image on the screen?

5. Find the radius of curvature of the given concave mirror.

6. Plot a curve showing, for the given concave mirror, how the distance of the image depends upon the distance of the object. Hence deduce the focal length of the mirror and also its radius of curvature.

7. Being given a convex lens, two pin points on stands, and a metre scale, make such observations as shall enable

you to verify the law expressing the relation between the distances of an object and its real image from the lens.

8. Plot a curve showing the relationship between the size of an image and its distance from the lens which forms the image.

9. Show that the magnification of an image formed by a concave mirror is directly proportional to v/u, where v and u are the distances of image and object respectively from the mirror.

10. Find the focal length of the lens formed by filling the given watch glass (*a*) with the liquid **A**, and (*b*) with the liquid **B**. State which of the liquids has the higher refractive index.

CHAPTER XV.

1. Two tuning-forks (a C-fork and a D-fork) were compared by the dropping-plate method. Using the symbols of Fig. 109 (p. 165), the following data were obtained:

(i) (with C-fork).	(ii) (with D-fork).
$\mathbf{O}a = 0{\cdot}52$ cm.	$\mathbf{O}a = 0{\cdot}36$ cm.
$\mathbf{O}b = 15{\cdot}35$,,	$\mathbf{O}b = 16{\cdot}14$,,
$n = 39$	$n = 47$

Calculate the frequency of each fork, and determine to what extent the ratio of these differs from 9/8.

2. In an experiment to determine the frequency of a tuning-fork, by the sonometer method, the following data were obtained:

Stretching force (**F**), - - -	6 kilograms.
Length of wire in unison with fork,	58·35 cm.
,, ,, cut off for weighing,	57·4 ,,
Weight of this wire, - - -	0·3552 gm.

From the equation $n = \frac{1}{2l}\sqrt{\frac{\mathbf{F}}{m}}$ calculate the frequency of the fork.

3. The frequency of transverse vibration of a stretched string is increased from 300 to 400 by adding 7 kilograms to the stretching force. What additional force would be required to raise the frequency to 600?

4. Compare the frequencies of vibration of the two given tuning-forks by two methods.

5. Compare the frequencies of two given tuning-forks by the sonometer.

6. Find what lengths of the given resonance tubes of different diameters resound to the given fork.

CHAPTER XVI.

1. A very long magnet produces a magnetic force of 1 dyne per unit pole at a point on its axis distant 10 cm. from its N-seeking pole. What alteration in this force would occur if the S-seeking pole, instead of being far away, occupied a position 5 cm. from the N-seeking pole?

2. In a rough determination of the horizontal component (**H**) of the earth's magnetic field, by means of a rectangular bar-magnet and a magnetometer, the following data were obtained:

(i) **M/H**. The bar-magnet was placed in the *end-on* position.

Magnetic length ($2l$) of bar-magnet, -	11·4 cm.
Distance (d) from centre of magnet to magnetometer needle, - - -	25 cm.
Mean deflection (θ), - - - -	39°·85

Calculate the value of M/H by means of the equation

$$\frac{M}{H}=\frac{(d^2-l^2)^2}{2d}.\tan\theta.$$

(ii) **MH**.

Length of bar-magnet, - -	12·7 cm.
Breadth " " - -	1·437 cm.
Weight " " - -	87·94 gm.

Calculate the *moment of inertia* (**I**) of the magnet by the equation :

$$I = \frac{(\text{length})^2 + (\text{breadth})^2}{12} \times \text{weight}.$$

The magnet was then suspended horizontally, and

62 oscillations occupied 17 min. 16·4 sec.

Calculate the value of **MH** by the equation

$$t = 2\pi\sqrt{I/MH}.$$

(iii) From the values obtained for **M/H** and **MH** calculate that of **H**.

3. Deflect a small balanced magnet by a bar-magnet lying magnetic E. and W., and directed towards its centre at various distances from it, and plot a graph showing the relation between the distances separating the centres of the magnets and the deflections.

4. Place two large magnets east and west, with their S-seeking poles facing each other, and 30 cm. apart. Compare the intensity of the field at a point midway between the poles, and at a point on the same level 30 cm. north-west of the first.

5. Compare the values of the magnetic force due to the given bar-magnet at points on its axis 10 and 15 cm. beyond one end.

6. Compare the intensities of the horizontal component of the earth's magnetic field at different places by the method of vibrations.

7. Plot a curve showing the relationship between the distance of a magnet (in the end-on position) and the deflection produced in a deflection magnetometer.

8. Find the horizontal intensity of the earth's magnetic field in the laboratory.

CHAPTER XVIII.

1. Calculate the specific resistance of 'eureka' alloy from the following data:

Length of wire, - - - - -	99·9 cm.
Diameter, - - - - -	0·045 ,,
Resistance, - - - - -	2·907 ohm.

2. In a comparison of the electromotive forces of a Daniell cell and a Leclanché cell, by the method of sum and difference, the following observations were obtained:

With the cells in opposition, mean deflection $= 8^\circ{\cdot}7$.
,, ,, ,, conjunction, ,, ,, $= 50^\circ{\cdot}4$.

Assuming that the E.M.F. of the Daniell cell is 1·10 volt, calculate that of the Leclanché cell.

3. The following data were obtained in a measurement of the resistance of a bronze telegraph wire (as used by the General Post Office). Calculate the *specific resistance* of the material.

Length of wire, - - - - -	488·6 cm.
Diameter, - - - - - -	1·267 mm.
Resistance, - - - - -	0·13 ohm.

4. In a determination of the reduction-factor of the low-resistance coil of a tangent galvanometer, by electrolysis of copper sulphate, the following data were obtained:

Initial weight of cathode, - -	11·337 gm.
Time, at closing of the circuit, -	4 h. 48 min.
,, ,, breaking ,, ,, - -	5 h. 8 min.
Final weight of cathode, - -	11·563 gm.
Mean deflection of galvanometer, -	$31^\circ{\cdot}27$

If the electro-chemical equivalent of copper is 0·000329, calculate the reduction-factor of the galvanometer.

5. The reduction-factor of the coil used in Exercise 4 was determined by measuring the heat developed in a

spiral of manganin wire immersed in a known weight of water. The following data were obtained:

Resistance of spiral, - - -	21·54 ohms.
Weight of copper calorimeter, -	53·25 gm.
" " calorimeter + water, -	235·2 "
Initial temperature of water, - -	17°·2 C.
Time, at closing of the circuit, -	8 h. 52 min.
" " breaking " " -	9 h. 10 min.
Final temperature of water, - -	20°·8 C.
Mean deflection of galvanometer, -	20°·6

If the specific heat of copper is 0·095, and the mechanical equivalent of heat is $4{\cdot}2 \times 10^7$ ergs per calorie, calculate the reduction-factor of the galvanometer.

6. Arrange two Daniell's cells (1) in series, (2) in parallel with a tangent galvanometer and a resistance box. Measure the current given by each arrangement, first with no resistance, then with a resistance of 20 ohms in the box, and comment upon the results.

7. Compare the advantages of placing two cells in series and in parallel, first using the high-resistance coil and an additional resistance of 100 ohms, and secondly, the low-resistance coil and no additional resistance.

8. Compare the advantages of using the low-resistance coil of the galvanometer over the high-resistance coil for measuring resistances by the method of substitution.

9. Plot a curve showing the relationship between the resistance in the circuit and the tangent of deflection in a tangent galvanometer. Neglect the resistances of the battery and of the galvanometer.

10. Construct a simple stretched-wire "bridge" out of the material supplied, and use it to find the ratio of the resistances of equal lengths of the wires P and Q.

11. The given tangent galvanometer has two separate coils of equal radius. Compare the deflections produced when the same current is sent (1) in the same direction,

(2) in opposite directions, through the coils. Hence deduce the number of turns in one coil, being given the number of turns in the other.

12. Find how the current through the given tangent galvanometer and battery depends upon the resistance included in series with them. Plot your results.

13. Compare the resistance of two given conductors by the Post Office Box.

14. Find how the moment of the given electromagnet depends upon the strength of the current passing through it.

15. Find the length of wire in the given resistance coil, being given a sample of the wire of which it is made.

16. Find the number of turns of wire in the galvanometer coil by observing the deflection for a known current.

17. Find the temperature coefficient of resistance of the given wire.

18. Compare the resistances of the two given coils by measuring the amount of heat generated in them by a current.

19. Compare the E.M.F.'s of the two given cells by a tangent galvanometer.

ANSWERS TO EXERCISES.

CHAPTER I.

1. 17·8 cm. **2.** 83° 52′. **3.** 29·53.

CHAPTER II.

5. $1{\cdot}03 \times 10^6$ c. ft.; $0{\cdot}78 \times 10^6$ c. ft.

CHAPTER IV.

1. 5·6576(5) gm.; 0·22 % too light. **2.** 1·656 mm.

CHAPTER V.

1. 14·7 lb. per sq. in.; 1033 gm. per sq. cm.
2. 80·95 cm. **3.** 2·16 c.c.

CHAPTER VI.

1. 2·62. **2.** $\frac{1}{1{\cdot}29}$; 115 gm. **3.** 0·75.
4. 8·0. **5.** 2·51; 1·080. **6.** 0·802.

CHAPTER VII.

1. 1·84 lb. and 0·77 lb. **2.** 980 lb.; 101·8 lb. and 1018 lb.
3. $\frac{1}{3}$ of length of either rod from both BC and CD.
4. 0·19 inch. **5.** $\dfrac{a^2c(\rho - 1)}{b^2 + a^2(\rho - 1)}$.

CHAPTER IX.

1. 0·000361 ; 0·000386. **2.** True coefficient = 0·00052.
3. $\frac{1}{357}$. **4.** 98·23 gm. **5.** 1·0001·2 sec. **6.** $-74^\circ{\cdot}6$ C.

CHAPTER X.

1. 0·093. **2.** 0·43. **3.** 517 calories. **4.** 0·25.

CHAPTER XII.

1. $4{\cdot}15 \times 10^7$ ergs. **2.** $4{\cdot}2 \times 10^7$ ergs. **3.** $3{\cdot}36 \times 10^{10}$ ergs.

CHAPTER XIII.

1. 36 %.

CHAPTER XIV.

1. $-24{\cdot}69$ cm. **2.** $-24{\cdot}3$ cm.; $+33{\cdot}4$ cm.
4. 80 cm., or 20 cm., from source.

CHAPTER XV.

1. 9/7·98. **2.** 264·3. **3.** 27 kilos.

CHAPTER XVI.

1. Reduced by $\frac{4}{9}$ dyne. **2.** 0·169.

CHAPTER XVIII.

1. $46{\cdot}3 \times 10^{-6}$ ohm. **2.** 1·42 volt.
3. $33{\cdot}52 \times 10^{-6}$ ohm. **4.** 0·94. **5.** 0·928.

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